Effects Of Peristaltic Flow Of A Couple Stress Fluids Through A Porous Medium In A Channel At Low Reynolds Number

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ABSTRACT
The present paper discuss the effects of the peristaltic motion of a couple stress fluid through a porous medium in a two dimensional flexible channel under long wave length approximation and negligible Reynolds number. The effects of various physical parameters on axial velocity, transverse velocity, pressure gradient and friction force have been computed numerically by perturbation method. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity, transverse velocity and pressure also. It is noted that pressure rise increases with increase in couple stress parameter and porous medium. The friction force has an opposite behavior compared with pressure rise.

Keywords: Peristaltic motion, Couple stress fluid, Porous medium and Reynolds number

1 INTRODUCTION
Peristalsis is an important mechanism for mixing and transporting physiological fluids, which is generated by the passage of progressive waves of area contraction and expansion over flexible walls of a tube containing fluid. This mechanism is found in many physiological situations like urine transport from kidney to the bladder through the ureter, swallowing food through the esophagus, movement of chyme in the gastrointestinal tract, transport of spermatozoa in the ducts efferentes of the male reproductive organ, movement of ovum in the female fallopian tube, vasomotion of small blood vessels, motion of spermatozoa in cervical canal, transport of bile in bile duct. Some worms like earth-worm use peristalsis for their locomotion. Some biomedical instruments such as heart-lung machine work on this principle. Mechanical devices like finger pumps, roller pumps use peristalsis to pump blood, slurries and corrosive fluids. It is also speculated that peristalsis may be involved in the translocation of water in tall trees. The translocation of water involves its motion through the porous matrix of the trees. The peristaltic transport of a toxic liquid is used in nuclear industry so as not to contaminate the outside environment.

Various studies on peristaltic transport, experimental as well as theoretical, have been carried out by many researchers to explain peristaltic pumping in physiological systems. The first attempt to study the fluid mechanics of peristaltic transport is done by Latham (Latham 1966). Burns and parkes (Burns and parkes 1967) discussed the peristaltic flow produced by sinusoidal peristaltic wave along a flexible wall of the channel under the pressure gradient and Shapiro and Jaffrin et al. (Shapiro and Jaffrin et al.1969) have studied inertial free peristaltic flow with long wavelength at low Reynolds number. The early developments of mathematical modeling and experimental fluid mechanics of peristaltic flow were given in a comprehensive review by Jaffrin and Shapiro 1971. After these studies, several authors Fung and Yih...
(Fung and Yih 1968), Zien and Ostrach (Zien and Ostrach 1970), Weinberg and Eckstein et al. (Weinberg and Eckstein et al. 1971), Raju and Devanathan (Raju and Devanathan 1972), Srivastava and Srivastava (Srivastava and Srivastava 1983), Elshehawey and Sobh (Elshehawey and Sobh 2001), Maiti (Maiti 2005), Radhakrishnamacharya and Srinivasulu (Radhakrishnamacharya and Srinivasulu 2007) have studied peristalsis under different conditions.

The most physiological fluids including blood behave as non-Newtonian fluids. Hence, the study of peristaltic transport of non-Newtonian fluids may help to get better understanding of the biological systems. Several researchers studied peristaltic transport of non-Newtonian fluids Radhakrishnamacharya (Radhakrishnamacharya 1982), Sobh and Al Azab et al. (Sobh and Al Azab et al.2010), Raghunatha Rao and prasada Rao (Raghunatha Rao and Prasada Rao 2011).

The study of couple stress fluid is very useful in understanding various physical problems because it possesses the mechanism to describe rheological complex fluids such as liquid crystals and human blood. By couple stress fluid, we mean a fluid whose particles sizes are taken into account, a special case of non-Newtonian fluids. In further investigation many authors have used one of the simplification is that they have assumed blood to be a suspension of spherical rigid particles (red cells), this suspension of spherical rigid particles will give rise to couple stresses in a fluid. The theory of couple stress was first developed by Stokes (Stokes 1966) and represents the simplest generalization of classical theory which allows for polar effects such as presence of couple stress and body couples. A number of studies containing couple stress have been investigated by Valanis and Sun (Valanis and Sun 1969), Chaturani (Chaturani 1978), Chaturani and Upadhya(Chaturani and Upadhya 1978), Chaturani and Rathod (Chaturani and Rathod 1981), (Srivastava (Srivastava 1986), Elshehawey and Mekheimer (Elshehawey and Mekheimer 1994), Elshehawey and El-Sebaei (Elshehawey and El-Sebaei2001), Mekheimer(Mekheimer 2002),Devakar and Iyengar (Devakar and Iyengar 2008), Sobh (Sobh 2008), Ravikumar et al.(Ravikumar et al. 2010), Sohail Nadeem and Sofia Akram (Sohail Nadeem and Sofia Akram 2011),Alemayehu and Radhakrishnamacharya (Alemayehu and Radhakrishnamacharya 2011).

Flow through porous media has been of considerable interest in the recent years due to the potential application in all fields of Engineering, Geo-fluid dynamics and Biomechanics. For example study of flow through porous media is immense use to understand transport process in lungs, in kidneys, gallbladder with stones, movement of small blood vessels and tissues cartilage and bones etc. Most of the tissues in the body (e.g. bone, cartilage, muscle) are deformable porous media. The proper functioning of such materials depends crucially on the flow of blood, nutrients and so forth through them. Porous –medium models are used to understand various medical conditions (such as tumor growth) and treatments (such as injections).

The present research aimed is to investigate the interaction of peristalsis for the motion of a couple stress fluid through a porous medium in a two dimensional flexible channel at low Reynolds number under long wavelength approximation. A Perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity. The computational analysis has been carried out for drawing Velocity profiles, Pressure and Frictional force.

2 FORMULATION OF THE PROBLEM
We consider a peristaltic flow of a couple stress fluids through two-dimensional channel of width 2d, symmetric with respect to its axis. The walls of the channel are assumed to be flexible. The travelling waves are represented by (Figure 1)
\[ \psi(X, t) = \beta + \alpha \cos \left[ \frac{2\pi}{\lambda}(X^1) \right] \]  

Where ‘\( \alpha \)’ is the amplitude of the peristaltic wave, ‘\( c \)’ is the wave velocity, ‘\( \lambda \)’ is the wave length, \( t \) is the time period, \( X^1 \) is the direction of wave propagation.

The constitutive equations and equations of motion for a couple stress fluid are stokes (Stokes 1966)

\[ T_{ji,j} + \rho f_i = \rho \frac{\partial v_i}{\partial t}, \]  

(2)

\[ e_{ijk} T_{ij}^A + M_{ji,j} + \rho c_i = 0, \]  

(3)

\[ \tau_{ij} = -p\delta_{ij} + 2\mu \beta_{ij}, \]  

(4)

\[ \mu_{ij} = 4\psi^* w_{i,j} + 4\psi^{**} w_{i,j}, \]  

(5)

Where \( f_i \) is the body force vector per unit mass, \( c_i \) is the body moment per unit mass, \( v_i \) is the velocity vector, \( \tau_{ij} \) and \( T_{ij}^A \) are the symmetric and antisymmetric parts of the stress tensor \( T_{ji} \), respectively, \( M_{ij} \) is the couple stress tensor, \( \mu_{ij} \) is the derivative part of \( M_{ij} \), \( \omega_i \) is the vorticity vector, \( \beta_{ij} \) is the symmetric part of the velocity gradient, \( \mu \) is the viscosity of the fluid, \( \psi^* \) and \( \psi^{**} \) are constants associated with the couple stress, \( p \) is the pressure, and \( \delta_{ij} \) is the Kroneker delta.

Neglecting the body force and the body couples, the continuity equation and equations of motion Mekheimer (Mekheimer 2002) through a porous medium are

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(6)

\[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \nabla^2 u - \psi^* \nabla^4 u - \frac{\mu}{k_1} u \]  

(7)

\[ \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \nabla^2 v - \psi^* \nabla^4 v - \frac{\mu}{k_1} v \]  

(8)

Where \( \nabla^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right), \quad \nabla^4 = \nabla^2 \nabla^2 \)

\( u \) and \( v \) are velocity components, ‘\( p \)’ is the fluid pressure, ‘\( \rho \)’ is the density of the fluid, ‘\( \mu \)’ is the coefficient of viscosity, and ‘\( \psi^* \)’ is the coefficient of couple stress, \( K_1 \) is the permeability of the porous medium.

The relative boundary conditions are

\[ u = 0 \text{ at } y = \psi \]  

(9)

\[ \frac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = \psi \]  

(10)

\[ v = 0 \text{ at } y = 0 \]  

(11)
Equation (9) corresponds to no slip on the boundary, (10) indicates the boundary condition related to couple stress fluid and (11) indicates velocity at the centre of the channel.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

\[ x = X - ct, \quad y = Y, \quad u = u - c, \quad v = v, \quad p = P(X, t) \]

Using the following the non-dimensional variables

\[ x' = \frac{x}{\lambda}, \quad y' = \frac{y}{\beta}, \quad u' = \frac{u}{c}, \quad v' = \frac{v}{c\delta}, \quad t' = \frac{ct}{\lambda}, \quad \psi' = \frac{\psi}{\beta}, \quad p' = \frac{p\beta^2}{\mu c\lambda} \quad (12) \]

Substituting equation (12) in equations (1) and (6) to (11), these equations reduces to (after dropping primes)

\[ Y = \psi(x) = \pm 1 \quad (13) \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (14) \]

\[ \text{Re} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \left( \delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - K \left( \delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - D^{-1} u \quad (15) \]

\[ \text{Re} \delta^3 \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial p}{\partial y} + \delta^2 \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - K \delta^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - D^{-1} \delta^2 v \quad (16) \]

The corresponding dimensionless boundary conditions are

\[ u = -1 \quad \text{at} \quad y = \pm \psi \quad (17) \]

\[ \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = \pm \psi \quad (18) \]

\[ v = 0 \quad \text{at} \quad y = 0 \quad (19) \]

Where

\[ \epsilon = \frac{\alpha}{\beta}, \quad \delta = \frac{\beta}{\lambda} \] are the geometric parameters

\[ \text{Re} = \frac{c\beta}{\nu} \] is the Reynolds number

\[ S = \frac{\psi^*}{\beta} \] is the Couple stress parameter

\[ D = \frac{K_1}{\beta^2} \] is the porous parameter

\[ X' = X - ct \]

**3 METHOD OF SOLUTION**

We seek perturbation solution in terms of small parameter δ as follows:

\[ u = u_0 + \delta u_1 + \delta^2 u_2 + \ldots \quad (20) \]

\[ v = v_0 + \delta v_1 + \delta^2 v_2 + \ldots \quad (21) \]

Substituting equations (20) & (21) in equations (15) & (16) and collecting the coefficients of various powers of δ

The zeroth order equations are

\[ K \frac{\partial^4 u_0}{\partial y^4} - \frac{\partial^2 u_0}{\partial x^2} + D^{-1} u_0 = - \frac{\partial p_0}{\partial x} \quad (22) \]

\[ \frac{\partial p_0}{\partial y} = 0 \quad (23) \]

The corresponding boundary conditions are

\[ u_0 = -1 \quad \text{at} \quad y = \pm \psi \quad (24) \]
On solving the equations (22) subject to the conditions (24) & (25), we obtain
\[ u_0 = M_3 \cos h \; m_1 y - M_4 \cos h \; m_2 y - PD \] (27)

Using equation of continuity (2) and subject to the condition (26), we obtain
\[ v_0 = \frac{a_2}{m_2} g_3 \tan h \{m_1 \psi \} \sec h \{m_1 \psi \} \sin h \{m_1 y \} - \frac{a_1}{m_2} g_3 \tan h \{m_2 \psi \} \sec h \{m_2 \psi \} \sin h \{m_2 y \} \]

The equations corresponding to the order δ are
\[ K \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial^2 u_1}{\partial y^2} + D^{-1} u_1 = \frac{\partial p_1}{\partial x} - R \left( \frac{\partial u_0}{\partial t} + u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} \right) \] (29)

\[ \frac{\partial p_1}{\partial y} = 0 \] (30)

The corresponding boundary conditions are
\[ u_1 = 0 \quad \text{at} \quad y = \pm \psi \] (31)
\[ \frac{\partial^2 u_1}{\partial y^2} = 0 \quad \text{at} \quad y = \pm \psi \] (32)
\[ v_1 = 0 \quad \text{at} \quad y = 0 \] (33)

On solving the equation (29) subject to the conditions (31) & (32), we obtain
\[ u_1 = g_1 \cos h \; m_1 y + g_2 \cos h \; m_2 y - N_4 \cos h \{m_1 + m_2 \} y - N_4 \cos h \{m_1 - m_2 \} y + N_4 \cos h \{2m_1 \} y - N_4 \cos h \{2m_2 \} y + N_2 - N_6 - M_7 \] (34)

Using equation of continuity (2) and subject to the condition (33), we obtain
\[ v_1 = \left( \frac{\theta_{10} - \theta_{4s}}{m_1} \right) \sin h \{m_1 y \} - \left( \frac{\theta_{11} + \theta_{5s}}{m_1} \right) \sin h \{m_2 y \} - \frac{\theta_{10}}{m_2 y} \sin h \{2m_1 y \} + \frac{\theta_{4s}}{2m_2 y} \sin h \{2m_2 y \} + \frac{a_1}{m_1 + m_2} \sin h \{m_1 - m_2 \} y + \frac{a_1}{m_1 + m_2} \sin h \{(m_1 + m_2) y \} + (g_8 - g_6) y \] (35)

The volumetric flow rate in the wave frame is defined by
\[ q = \int_0^\psi u \mathrm{d}y = (L + H + I)P^2 + J \; P + G \] (36)

Where
\[ m_1 = \sqrt{\frac{1}{k} - \left( \frac{\psi}{2} \right)^2 - \frac{4}{D \psi}} \], \[ m_2 = \sqrt{\frac{1}{k} - \left( \frac{\psi}{2} \right)^2 - \frac{4}{D \psi}} \], \[ P = \frac{\partial P}{\partial x} \], \[ a_1 = \frac{m_1^2}{m_2^2 - m_1^2}, \quad a_2 = \frac{m_2^2}{m_2^2 - m_1^2} \], \[ M_1 = \frac{a_2}{\cos h \{m_1 \psi \}}, \quad M_2 = \frac{a_1}{\cos h \{m_2 \psi \}}, \quad M_3 = M_5 (PD - 1), \quad M_4 = M_6 (PD - 1), \quad M_5 = a_2 g_1 \tan h \{m_2 \psi \}, \quad M_6 = a_1 g_1 \tan h \{m_1 \psi \} \sec h \{m_1 \psi \}, \quad M_7 = \frac{\partial p_1}{\partial x} D, \quad N_1 = \frac{R D M_1 M_5}{2k \left( \frac{4m_1^4}{2} + \frac{1}{k} \right)}, \quad N_2 = \frac{R D M_1 M_5}{2} \], \[ N_3 = \frac{R (M_1 M_6 + M_2 M_5)}{2k \left( \frac{4m_1 m_2}{2} + \frac{1}{k} \right)} \], \[ N_4 = \frac{R (M_1 M_6 + M_2 M_5)}{2k \left( \frac{4m_1 m_2}{2} + \frac{1}{k} \right)} \].
\[ N_5 = \frac{RDM_2M_6}{2k(4M_2^2 - \frac{m_2^2}{x^2} + \frac{1}{D})}, \quad N_6 = \frac{R D M_2 M_6}{2} \quad N_7 = \frac{RPM_5}{2k(\frac{m_2^2 + m_1^2}{x^2} + \frac{1}{D})} \quad N_8 = \frac{RPM_5}{2k(\frac{m_2^2 + m_1^2}{x^2} + \frac{1}{D})}, \quad N_9 = -p_r \quad E \frac{m_1^2}{M_1^2} \quad N_{10} = -\frac{p_rE}{D} \frac{M_1^2}{M_1^2} \]

\[ B_1 = \frac{1}{(m_2^2 - m_1^2)} \cos h[m_1^2] \left( m_2^2 - 4m_1^2 - m_2^2 \right) M_{11} \cosh[2m_1 \psi] + m_2^2 M_{12} + m_2^2 + 2m_1 m_2 \right] M_{13} \cosh[(m_1 + m_2) \psi] + m_1^2 M_{14} \cosh[(m_1 - m_2) \psi] + 3m_1^2 M_{15} \cosh[2m_1 \psi] - m_2^2 M_{16} + (m_1^2 - m_2^2) M_{17} \cosh[m_1 \psi] \]

\[ B_2 = \frac{1}{(m_2^2 - m_1^2)} \cos h[m_2^2] \left( m_1^2 - 3m_2^2 M_{11} \cosh[2m_1 \psi] + m_1^2 M_{12} - m_2^2 M_{16} + (m_2^2 + 2m_1 m_2) M_{13} \cosh[(m_1 + m_2) \psi] + (m_2^2 - 2m_1 m_2) M_{14} \cosh[(m_1 - m_2) \psi] - (m_2^2 - m_2^2) M_{18} \cosh[2m_2 \psi] + (4m_2^2 - m_1^2) M_{15} \cosh[2m_1 \psi] \right)
\]

\[ g_1 = \frac{g_{1+PD}}{m_1}, \quad g_2 = \frac{g_{1-PD}}{m_2}, \quad g_3 = \psi_x, \quad g_4 = B_{1x}, \quad g_5 = B_{2x}, \quad g_6 = N_{2x} \]

\[ g_7 = N_{3x}, \quad g_8 = N_{4x}, \quad g_9 = N_{5x}, \quad g_{10} = N_{7x}, \quad g_{11} = N_{8x}, \]

\[ G = \delta \left( \frac{1}{m_2^2 - m_1^2} \left( m_1 \sech[m_2^2] \right) - \frac{1}{m_2^2} \sech[m_1 \psi] - \frac{1}{m_2} \sech[m_1 \psi] - 2 \right) \]

\[ H = \delta \left( \frac{\sinh[m_2^2]}{m_2} \right) \left( 3m_1^2 N_1 \cos h[2m_1 \psi] + m_2^2 N_2 + (m_2^2 + 2m_1 m_2) N_3 \cos h[(m_1 + m_2) \psi] + (m_2^2 - 2m_1 m_2) N_6 \cos h[m_2 \psi] \right) + \]

\[ N_{14} N_3 \left( m_1^2 - m_2^2 \right) \cos h[m_1 \psi] \left( m_1 \cosh[2m_1 \psi] \right) + \frac{N_4}{2m_1} \sin h[2m_1 \psi] + \frac{N_5}{2m_2} \sin h[2m_2 \psi] + \]

\[ \frac{N_6}{m_1 + m_2} \sin h[(m_1 + m_2) \psi] + \frac{N_4}{m_1 + m_2} \sin h[(m_1 - m_2) \psi] + (N_2 - N_6) \gamma, \]

\[ J = D \left( \frac{a_2 m_2}{m_1} \right) \sin h[m_1 \psi] \sin h[m_1 \psi] - (N_9 + N_{10}) \sec h[m_2 \psi] \sin h[m_2 \psi] - \gamma, \]

\[ L = \delta \left( \frac{\sinh[m_2^2]}{m_1} \right) \left( m_2^2 - 4m_1^2 \right) N_1 \cos h[2m_1 \psi] + m_2^2 N_2 - m_2^2 N_6 + \left( m_1^2 + 2m_1 m_2 \right) N_3 \cos h[(m_1 + m_2) \psi] + \left( m_2^2 - 2m_1 m_2 \right) N_4 \cos h[(m_1 - m_2) \psi] + 3m_2^2 N_5 \cos h[2m_2 \psi] - \left( m_2^2 - m_2^2 \right) N_7 \cos h[(m_1 + m_1) \psi] \right) \]

The instantaneous flux \( Q(x, t) \) in the laboratory frame is

\[ Q = f_0^y u + 1 \, dy = q + \psi \quad (37) \]

The average flux \( \bar{Q} \) over one period of peristaltic wane is

\[ \bar{Q} = \frac{1}{T} \int_0^T Q \, dt = q + 1 \quad (38) \]

From equations (36) and (38), the pressure gradient is obtained as

\[ \frac{dp}{dx} = \frac{-f_{f_0}^x (x - 4G + H + I)(x - 4G + H + I) + 1}{2(G + H + I)} \]

The pressure rise (drop) over one cycle of the wave can be obtained as

\[ \Delta p = \int_{f_0}^1 \frac{dp}{dx} \, dx \]

The dimensionless frictional force \( F \) at the wall across one wavelength is given by

\[ F = \int_{f_0}^1 \psi^2 \left( -\frac{dp}{dx} \right) \, dx \]
4 RESULTS AND DISCUSSIONS

In this section, we have presented the graphical results of the solutions axial velocity $u$, transverse velocity $v$, pressure rise $\Delta p$, friction force $F$. The axial velocity is shown in Figures (2) to (5), for the different values of $R$, $\varepsilon$, $\delta$, $S$, $D$. There is no appreciable effect of Reynolds number $R$, on the axial velocity $u$. The axial velocity $u$ is exhibit in figure (2) for a different values of amplitude ratio $\varepsilon$ in the region $y=0$ to $y=1$. It is found that the velocity $u$ is maximum on the central line $y=0$ and gradually decreases to attain the prescribed value $-1$ at $y=1$. An increase in $\varepsilon$ enhances the magnitude of $u$ in the flow region. Figure (3) represent the variation of $u$ with slope parameter $\delta$, we notice that higher the slope of the boundary lesser $|u|$ in the flow region. The Variation of $u$ with $S$ shows that $|u|$ experiences an enhancement with increase in $S$ (Figure4). The variation of $u$ with porous parameter $D$ shows that for smaller values of permeability with Porous medium, the variation of $u$ is insignificant in the region $y=0$ to $y=0.4$, and then $u$ gradually Falls $-1$ on $y=1$, and for higher permeability $u$ is almost linear in the flow region for still higher values of the permeability the magnitude $u$ slowly reduces to its maximum on $y=1$ to attain the prescribed value $u=-1$ on $y=1$ (Figure 5).

The variation of transverse velocity $v$ shows in Figures (6 to 9) for different values of $R$, $\varepsilon$, $\delta$, $S$, $D$. There is no appreciable effect of Reynolds number $R$, on the transverse velocity $v$. The Transverse velocity rises from its value 0 on the central line $y=0$ to attain the maximum value on the boundary on $y=1$, an increase in $\varepsilon$ leads to an enhancement in $v$ throughout the region in Figure (6). From Figure (7), we notice that higher the slope parameter $\delta$ of boundary wall, larger the velocity $v$ in the flow region. The Variation of $v$ with $S$, we find that $v$ depreciates with increase in $S$ (Figure8). From Figure (9), we find that for the smaller values of permeability $v$ changes from negative to positive at $y=0.2$ and again changes from positive to negative at $y=0.7$ and attains the max, at $y=1$, and for higher values of permeability we notice a marginal increase in $v$.

The variation of pressure rise $\Delta p$ against the average volume flux $\bar{Q}$ is shown in Figure (10 to 14) for different values of $R$, $\varepsilon$, $\delta$, $S$ and $D$. From Figures (10 to 13) we notice that for a different values of $R, \varepsilon, \delta,$ and $S$, the $\Delta p$ gradually falls down from its maximum value1 on $\bar{Q} =0$. As $\bar{Q}$ increases, $\Delta p$ gradually falls from the value 1 to 0.85. $\Delta p$ decreases with increases in $\bar{Q}$. An increase in $R$, $\varepsilon$, $\delta$, and $S$ results an enhancement in the $\Delta p$.The variation of $\Delta p$ is appreciably large for the smaller parametric values and it is marginal for higher parametric Values. From Figure (14), we find that higher the permeable porous medium, smaller the $\Delta p$.

The Frictional force $F$ is shown in Fig (15 to 19) for a different values of $R$, $\varepsilon$, $\delta$, $S$ and $D$. It is noticed that the variation of $F$ against the average volume flux $\bar{Q}$, gradually rises from -1.05 to attain the maximum at $\bar{Q} =1$, from this we notice that the Frictional force $F$ enhances with increase in $\bar{Q}$. An increase in $R$, $\varepsilon$ depreciates the Frictional force $F$ in the flow region (Fig 15& 16). From Figure (17), we notice that the higher the slope parameter $\delta$ of the boundary wall, larger the magnitude $F$ in the flow region. From Fig (18) the variation of $F$ with $S$ shows that $F$ fluctuates with increase in $S$. From Fig (19), we find that the permeable porous medium $D$ larger the $|F|$ and for still higher the permeable porous medium, the enhancement in $|F|$ is appreciably large in comparison with lower the permeable porous medium.
Figure 2: effect of $\varepsilon$ on $u$ when $R = 1$, $\delta = 0.01$, $S = 0.1$, $D = 1$

Figure 3: Effect of $\delta$ on $u$ when $R = 1$, $\varepsilon = 0.01$, $S = 0.1$,

Figure 4: Effect of $S$ on $u$ when $R = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, $D = 1$
Figure 5: Effect of D on u when R = 1, ε = 0.01, δ = 0.01, S = 1

Figure 6: Effect of ε on v when R = 1, δ = 0.01, S = 0.1, D = 1

Figure 7: Effect of δ on v when R = 1, ε = 0.01, S = 0.1, D = 1
Figure 8: Effect of S on v when R = 1, \( \varepsilon = 0.01 \), \( \delta = 0.01 \), D = 1

Figure 9: Effect of D on v when R = 1, \( \varepsilon = 0.01 \), \( \delta = 0.01 \), S = 1

Figure 10: Effect of R on \( \Delta p \) when \( \varepsilon = 0.01 \), \( \delta = 0.01 \), S = 0.1, D = 1
Figure 11: Effect of $\varepsilon$ on $\Delta p$ when $R = 1$, $\delta = 0.01$, $S = 0.1$, $D = 1$

Figure 12: Effect of $\delta$ on $\Delta p$ when $\varepsilon = 0.01$, $R = 1$, $S = 0.1$, $D = 1$

Figure 13: Effect of $S$ on $\Delta p$ when $\varepsilon = 0.01$, $\delta = 0.01$, $R = 1$, $D = 1$
Figure 14: Effect of D on $\Delta p$ when $\varepsilon = 0.01$, $\delta = 0.01$, $R = 1$, $S = 1$

Figure 15: Effect of R on F when $\varepsilon = 0.01$, $\delta = 0.01$, $S = 0.1$, $D = 1$

Figure 16: Effect of $\varepsilon$ on F when $R = 1$, $\delta = 0.01$, $S = 0.1$, $D = 1$
Figure 17: Effect of $\delta$ on $F$ when $\epsilon = 0.01$, $R = 1$, $S = 0.1$, $D = 1$

Figure 18: Effect of $S$ on $F$ when $\epsilon = 0.01$, $\delta = 0.01$, $R = 1$, $D = 1$

Figure 19: Effect of $D$ on $F$ when $\epsilon = 0.01$, $\delta = 0.01$, $R = 1$, $S = 1$
5 CONCLUSIONS
In this paper we presented a theoretical approach to study the peristaltic flow of a couple Stress fluid through a porous medium at low Reynolds number in a flexible channel. The Governing equations of motion are solved analytically by perturbation method using long wave length approximation. We discuss the effect of various values of parameters on axial velocity; pressure rise and Frictional force per wave length have been computed numerically and explained graphically. We conclude the following observations:

1. The effect of Reynolds number $R$ is negligible, on the velocity $u$ & $v$.
2. The magnitude of the axial velocity $u$ increases with increase in amplitude ratio $\varepsilon$ and Decreases with increase in slope parameter $\delta$ and couple Stress parameter $S$.
3. For higher permeability $D$, $u$ is almost linear in the flow region for a still higher value of The permeability the magnitude $u$ is slowly reduces.
4. The transverse velocity $v$ increases with increase in amplitude ratio $\varepsilon$ and slope parameter $\Delta$ decreases with the increase in couple Stress parameter $S$.
5. Pressure rise $\Delta p$ decreases with increase in couple Stress parameter $S$ and permeable Porous medium $D$.Pressure rise $\Delta p$ increases with the increasing velocity.
6. An increase in Reynolds number $R$, amplitude ratio $\varepsilon$ and slope parameter $\delta$ results an Enhancement in the $\Delta p$.
7. The friction force $F$ has an opposite behavior compared with pressure rise $\Delta p$.

REFERENCES