



IJETST
Open access Journal

International Journal of Emerging Trends in Science and Technology

IC Value: 76.89 (Index Copernicus) Impact Factor: 4.219 DOI: <https://dx.doi.org/10.18535/ijetst/v4i11.07>

Convection, Segregation and Pattern Formation in Horizontally Vibrated Granular Materials

Authors

Abdul Rehman¹, Ping Wu^{1*}, Sana Shoukat² and Li Wang³

¹School of Mathematics and Physics, University of Science & Technology Beijing, Beijing, 100083, China

²College of Biological Sciences and Biotechnology, Beijing Forestry University, 100083, Beijing China

³School of Energy and Environmental Engineering, University of Science & Technology Beijing, Beijing, 100083, China

Abstract

We reviewed the developments in segregation, convection and pattern formation in horizontally vibrated granular systems which are important for the improvement of several industrial developments, such as agriculture, mining, pharmaceutical, geo-sciences, civil engineering etc. The most of previous theoretical and experimental research works were focused on vertical vibration direction; only limited studies explored the horizontally vibrated granular systems. Segregation, convection and pattern formation in horizontally vibrated systems are complex phenomena. Size ratio, particle properties, energy input, vibrational direction and boundary conditions are all playing significant roles. Several studies have been performed but extensive experiments and theoretical models are still mandatory to attain the firm explanations of these phenomena. Progresses in simulation software, novel analytical models and innovative experimental methods are desirable to sustain developments on these issues.

Keywords- Convection, Segregation, Pattern Formation, Stripe Formation, Granular Physics

1. Introduction

Assemblages of numerous jointly macroscopic particles are recognized as granular materials. Granular materials are everywhere around us and playing active roles in manufacturing and natural practices ^[1-3]. Granular materials are recognized as exciting phenomena which exhibit several miscellaneous physical conditions and unexpected dynamical occurrences ^[4]. Vibrational system's study is of direct application to several industrial procedures, such as drying ^[5], mixing of different materials ^[6], separation of valued elements from electronic leftover ^[7], and production of dampers used in areas extending from construction industry to aerospace engineering ^[8].

Energy can be delivered to a granular system in different manners such as horizontal direction vibration, vertical direction vibration, two direction vibration, three-dimensional vibration and other types of vibrations ^[9-13]. The vibrational direction shows influences on convection motion, which can affect the separation by size and pattern formations.

The dominant part of past investigations is in the vertical vibration direction.

The granular convection and segregation of binary or other systems in vertical shaking are extensively discoursed ^[14-17] and granular convection always remains significant for the segregation patterns ^[18-24]. Different convection patterns are observed due to heaping, ^[25, 26], tilting, ^[25, 27] etc. in vertically vibrated systems. Segregation in vertically vibrated granular systems also reveals extensive range of interesting phenomena ^[28-36]. A variety of segregation patterns ^[37-47] are also observed in vertically vibrated systems, such as stripe patterns ^[46] wave-like patterns ^[27], BNE and RBNE ^[34, 47] etc.

Only limited studies explored the effects of horizontal direction vibration. The phenomena of segregation and pattern formation are also observed in horizontally vibrated systems ^[48, 49]. T. Mullin ^[49] observed that random forces of one size particles to other size particles were one of causes of segregation patterns in horizontal vibrated binary

mixtures. P.M. Reis *et al.* ^[50, 51] observed three qualitative segregation phases - binary gas, segregation crystal and segregation liquid- in a horizontally vibrated binary sized particles system. Filling fraction of particles affected these phases significantly. The horizontal versions of BNE and RBNE were also observed in a horizontally vibrated circular particles system. Low particles density close to walls and high particles density at centre of the container are important for the phenomena ^[52].

Phenomena of convection also observed in horizontally vibrated granular materials ^[20, 53, 54]. K. Liffman *et al.* ^[54] observed two small convection rolls on the layer surface and two large lower internal convection rolls in a horizontally vibrated mono-sized particles system. M. Medved *et al.* ^[53] also observed convection rolls experimentally in a horizontally vibrated system, where friction of the side walls and the container size affect the shape and convection rolls significantly. Further researches were also suggested by M. Medved *et al.* ^[53] to understand the mechanism behind these convection rolls.

The relationships among convection, segregation and pattern formation are not available in previous literature. Therefore, extensive efforts are required to understand the complexity in phenomena of segregation ^[55-59], clustering ^[57, 60, 61], convection ^[12, 19, 62-65], and pattern formation ^[43, 61, 66] in horizontally vibrated granular systems at fundamental level. In this paper, we review the progresses in phenomena of convection, segregation and pattern formation in horizontally vibrated granular systems.

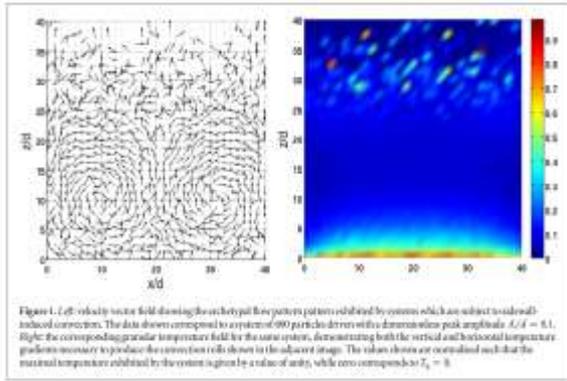
2. Convection In Horizontally Vibrated Granular Materials

Convection motion in horizontally vibrated granular systems presents complicated phenomena ^[12, 54, 67-69], and it remains an important subject for the recent investigations to understand the mechanics behind the segregation and pattern formation. ^[20, 70-73].

Convective motions detected inside granular systems are generally characterized into types of frictionally-driven convection ^[74, 75], buoyancy-driven convection ^[76, 77], and substrate-mediated convection ^[78]. R. Ramírez *et al.* ^[77] observed thermal convection in a square box by means of dynamics simulation. The existence of thermal convection is due to dissipative collisions between the particles and gravity. An Navier-Stokes granular hydrodynamics simulation showed that thermal

convection depends on inelastic collisions of particles ^[79]. B. Painter *et al.* ^[78] observed substrate-mediated type convection in a horizontally vibrated monolayer of hard particles system. They also observed four regimes in the system: solid-inclined regime, solid-flat regime, convective regime and disordered regime. The behaviors of granular convection in horizontally vibrated granular systems are fundamentally different from vertical vibrated systems ^[67]. Several types of convection rolls were observed in horizontally vibrated systems. Two convection rolls were observed in a horizontally vibrated rectangular vessel when particles ascended at the cell centre and plunged along the vertical walls ^[80]. J. Gallas *et al.* ^[81] also observed convection rolls in a horizontally vibrated system where convection speed was important for various kinds of movements. Four rolls convection were observed in horizontally oscillated granular materials: two small convective rolls on particle layer surface and two lower internal rolls. Particle layer surface sloshing is one of the important factor for small rolls and avalanche is important for lower internal rolls ^[54]. Three types of convection rolls, such as lower-right diagonal convection roll, homogeneous convection roll and upper-right diagonal convection roll and particles movements were observed experimentally in a horizontally vibrated square container. These convection rolls were influenced by heap position and vibrating parameters ^[12]. 2D molecular dynamics simulations of a horizontally vibrated granular materials showed that normal damping co-efficient affected the convection rolls: at high damping upper rolls were created, and at low damping lower rolls descended and disappear ^[82]. Several different states were observed in the horizontally vibrated system, such as 2D gas state, crystalline state and 3D gas state, depending on driving parameters. M. Hechel *et al.* ^[83] observed transitions between gas and crystalline states in horizontally vibrated systems. Transitions of states named as hysteresis depend on several factors, e.g. energy dissipation, and gapes between particles. T. Pöschel *et al.* ^[84] observed a slow inflation phase and a fast collapse phase in horizontally vibrated systems. Mechanical stability of materials and convection motion are playing important roles in alternation of slow inflation and fast collapse phase. Four regions namely the solid region, cluster region, gassy region and empty region and a 3D state (when particle jump from one of the corners of the container) were observed in a

horizontally vibrated mono-size particle system. Moment of inertia showed a strong effect on convection motion and energy dissipation of particles^[85]. The friction of the walls and the size of the container are also key factors for shape and number of convection rolls in horizontally vibrated systems^[53]. A pattern of convection in a horizontally system is shown in figure1^[86].



Earlier experiments^[87] and simulations^[15] exposed the connection between dissipative properties of the perpendicular walls and sidewall-induced convections. Dissipative walls remove energy rapidly from particles to increase the rate of convection motion^[86]. It is clear from the earlier efforts that inelastic collision, friction of side walls, buoyancy force, particles speed and particles densities between the centre and side walls of horizontally vibrated systems can lead strong convection motions. However, A comprehensive model for convection of horizontally vibrated granular materials is still not available. Therefore, further research works are required for better understanding the phenomena.

3. Segregation In Horizontal Vibrated Materials

When an originally mixed granular mixture grows into separate components forming clusters, granular segregation is observed. Segregation phenomena are influenced by apparent properties of particles, density, size and rigidity^[88-91]. Progresses in the research on granular segregation are important for industrial development, such as geophysics^[92-94], agriculture engineering^[95] and other industries^[96, 97]. The main objective in numerous industrial practices is to attain particle mixing, not separating or forming particles clusters^[88, 98]. Segregation phenomena have been described in both horizontally and vertically vibrated granular systems^[99, 100], avalanching^[101, 102], rotating drums

^[103], chute flows^[104, 105] and evacuating and filling of vessels^[106] etc.

Particles segregations were observed in horizontal vibrated granular systems of various sizes and properties of particles^[49, 90]. T. Mullin^[40] observed a series of perpendicular segregation patterns when the random forces of one size particles to other sized particles produced stripes patterns. Patterns coarsen with respect to time showed power law which is as the same as stone striping process in nature. A. Kudrolli *et al.*^[107] observed clusters in a 2D dense particles system. When the clusters appeared in the system, particles velocities decreased and the numbers of collisions between the particles increased. P. Reis and T. Mullin^[50] defined the combine filling fraction of the particles layer and explained that combined filling fraction C was significant for segregation patterns in the horizontally vibrated binary layers. When the critical value of filling fraction was $C_c = 0.647$ segregation was observed in the system. P. Reis *et al.*^[51] also observed phase evolution, such as segregation liquid, segregation crystal, and binary gas, in a horizontal vibrated granular system. Low filling fraction produced binary gas, intermediate filling fraction produced segregation liquid and high filling fraction produced segregation crystals. segregation by friction mechanism was explained by L. Kondic *et al.* in a horizontally vibrated mixture of highly smooth and rough particles^[29]. High coefficient of friction increased the rate of segregation in system. G. Ehrhardt *et al.*^[108] developed a numerical model for a horizontally vibrated binary granular mixture. The model explained that segregation rate increased with filling fraction. Horizontal versions of BNE and RBNE were also observed in a horizontally vibrated system, which depended on the ratio of the density to the size of the particles^[52]. The existence of convection can raise segregation patterns. Segregation can be controlled by altering the convection strength^[109]. C.R.K Windows-Yule *et al.*^[110] investigated the influence of convection on segregation in a binary system and determine that segregation rate increased with convection intensity. Various segregation states or phases are observed in horizontally vibrated granular systems, but the mechanism of segregation is still not clear^[111]. Theoretical models explaining under which conditions granular materials will be segregated in horizontally vibrated systems are also not available.

4. Pattern Formation In Horizontally Vibrated Granular Materials

When granular particles with different physical properties and masses are vibrated horizontally, a variety of patterns are observed in the systems^[43, 61, 66]. Stripe patterns are one of the most intriguing characteristics that have been discovered in horizontally vibrated systems^[49-51, 108, 112-116]. Stripe patterns can be formed due to linear and circular horizontal vibrations^[117]. In previous experiments, patterns were observed at very small and large frequencies and vibrational amplitudes^[37, 116, 118] or certain density level^[50, 108, 116]. P. Sánchez *et al.*^[118] observed stripe patterns in a liquid immersed, horizontally vibrated mixture of binary particles system. Their investigation suggested that stripe patterns were influenced by driving parameters, particles size and particles density. D. Kregel *et al.*^[37] observed stripe patterns in a horizontally vibrated system and found that these patterns were influenced by dissipative properties of particles. Direction shift of stripes was also observed in horizontally vibrated systems, which influenced by the container size and vibration parameters significantly. M. Fujii *et al.*^[114] observed direction alteration of stripes in horizontally vibrated binary particles system. At first, perpendicular stripes were observed in the system, after collisions between active large and small particles, the perpendicular stripes converted into parallel stripes. Stripe patterns also possess different qualitatively phases, segregation liquid, segregation crystal, and binary gas, in horizontal vibrated granular systems with respect to time scale^[48-51, 92, 116]. M.P. Ciamarra *et al.* observed stripe patterns by means of molecular dynamics simulation and proposed that large and small particles area fraction was significant for segregation process and pattern formation^[112]. R. Moosavi *et al.* observed a branched type stripe patterns at specific frequency and vibrational amplitude in a horizontally vibrated system and they purposed further investigation to examine the branches properties in the branched stripe pattern^[119]. Examples of stripe patterns are shown in Figure 2^[90]

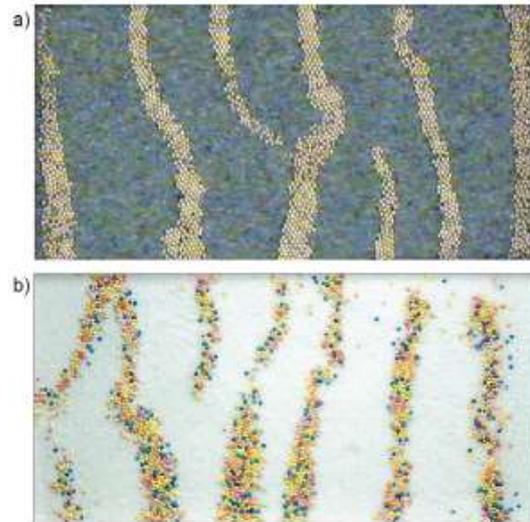


Figure 2 Typical segregation patterns of two binary granular mixtures: (a) poppy seeds (grey regions) + phosphor-bronze spheres (yellow regions); (b) polystyrene spheres (white regions) + sugar particles (coloured regions). The frames were captured after 15min of vibration of an initially homogeneous mixture, $C = 1:028$, $r = 2$.

Several experimental efforts have performed to understand convection, segregation and pattern formation in horizontally vibrated systems previously, but theoretical models are still not available. One subject received relatively little direct attention is the effects of convection on pattern formation in horizontally vibrated systems. These connections are essential for better understanding of these phenomena. Therefore, more innovative experimental techniques and latest simulation software are required to make theoretical models for convection and pattern formation in horizontally vibrated systems.

5. Conclusion

Granular materials behaviour is different from other usual and noticed types of materials. Most of the past investigations were focused on vertical vibration direction; limited studies explored the horizontally vibrated systems. Granular convection is essential for segregation and pattern formation in horizontally vibrated systems. Different convection patterns were observed in horizontally vibrated granular systems. Friction of the walls, the shape or size of the container, and number of convection rolls etc. significantly influence these convection patterns. Convection rolls show strong influences on segregation and pattern formation. Various types of

clusters and segregation patterns were also observed in horizontally vibrated systems, which showed enormous dependence on density, velocity, filling fraction of particles, friction between the particles and pressure variation in the systems. Some of previous works suggested that the segregation could be controlled by altering the convection strength and friction between the particles and walls.

Pattern formations observed in horizontally vibrated systems were influenced by solid-liquid phase transition, vibrating frequency, vibrating amplitude, tray size, and frictional contact of particles with the surface of container etc. Different qualitatively phases with respect to time scale are observed in horizontally vibrated system. Directional variation of stripes patterns and branched typed stripes were also observed in horizontally vibrated systems. Several efforts have been performed to understand convection, segregation and pattern formation in horizontally vibrated system. Considerable works are still required for development of theoretical models to understand these phenomena. Therefore, Progresses in simulation software, analytical models and innovative experimental methods are desirable to sustain development on these issues.

Acknowledgments

National Natural Science Foundation of China (51476009) sponsored this article

References

- [1] Rosato A D, Blackmore D L, Zhang N, Lan Y. A perspective on vibration-induced size segregation of granular materials [J]. *Chemical Engineering Science*, 2002, 57(2): 265-275.
- [2] Muzzio F J, Shinbrot T, Glasser B J. Powder technology in the pharmaceutical industry: the need to catch up fast [J]. *Powder Technology*, 2002, 124(1-2): 1-7.
- [3] Khakhar D V, Orpe A V, Hajra S K. Segregation of granular materials in rotating cylinders [J]. *Physica A: Statistical Mechanics and its Applications*, 2003, 318(1-2): 129-136.
- [4] Jaeger H M, Nagel S R, Behringer R P. Granular solids, liquids, and gases [J]. *Reviews of modern physics*, 1996, 68(4): 1259.
- [5] Liu C, Wang L, Wu P, Xiang F. Size distribution in gas vibration bed and its application on grain drying [J]. *Powder Technology*, 2012, 221: 192-198.
- [6] Zik O, Stavans J. Self-Diffusion in granular flows [J]. *EPL (Europhysics Letters)*, 1991, 16(3): 255.
- [7] Mohabuth N, Hall P, Miles N. Investigating the use of vertical vibration to recover metal from electrical and electronic waste [J]. *Minerals Engineering*, 2007, 20(9): 926-932.
- [8] Bannerman M N, Kollmer J E, Sack A, Heckel M, Mueller P, Pöschel T. Movers and shakers: Granular damping in microgravity [J]. *Physical Review E*, 2011, 84(1): 011301.
- [9] An X, Yang R, Dong K, Yu A. DEM study of crystallization of monosized spheres under mechanical vibrations [J]. *Computer Physics Communications*, 2011, 182(9): 1989-1994.
- [10] An X Z, He S S, Feng H D, Qian Q. Packing densification of binary mixtures of spheres and cubes subjected to 3D mechanical vibrations [J]. *Applied Physics A*, 2015, 118(1): 151-162.
- [11] An X, Li C, Qian Q. Experimental study on the 3D vibrated packing densification of binary sphere mixtures [J]. *Particuology*, 2016, 27: 110-114.
- [12] Abdul R, Wu P, Li L, Zhang S, Wang L. Convection rolls and individual particles movements in horizontally vibrated granular particles System [J]. *Acta Physica Polonica A*, 2016, 130: 1336.
- [13] Li L, Wu P, Abdul R, Wang L, Zhang S, Xie Z-A. Energy-dissipation correlated size separation of granular matter under coupling vibration and airflow [J]. *Powder Technology*, 2017, 307: 84-89.
- [14] Yan X, Shi Q, Hou M, Lu K, Chan C K. Effects of air on the segregation of particles in a shaken granular bed [J]. *Physical Review Letters*, 2003, 91(1): 014302.
- [15] Talbot J, Viot P. Wall-enhanced convection in vibrofluidized granular systems [J]. *Physical Review Letters*, 2002, 89(6): 064301.
- [16] Hou W, Zhang S, Wu P, Li L, Wang L. Effects of vertical vibration on surface intruder loading in a multiple-size granular

- system [C]. *Powder & Grains* 2017, 2017, 140(05006(2017)):
- [17] Liu C, Wang L, Wu P, Jia M. Effects of gas flow on granular size separation [J]. *Physical Review Letters*, 2010, 104(18): 188001.
- [18] Ehrichs E E, Jaeger H M, Karczmar G S, Knight J B, Kuperman V Y, Nagel S R. Granular Convection Observed by Magnetic Resonance Imaging [J]. *Science*, 1995, 267(5204): 1632-1634.
- [19] Knight J B, Ehrichs E E, Kuperman V Y, Flint J K, Jaeger H M, Nagel S R. Experimental study of granular convection [J]. *Physical Review E*, 1996, 54(5): 5726-5738.
- [20] Hsiau S-S, Ou M-Y, Tai C-H. The flow behavior of granular material due to horizontal shaking [J]. *Advanced Powder Technology*, 2002, 13(2): 167-180.
- [21] Lu L-S, Hsiau S-S. Mixing in a vibrated granular bed: Diffusive and convective effects [J]. *Powder Technology*, 2008, 184(1): 31-43.
- [22] Zeilstra C, Collignon J, Van der Hoef M, Deen N, Kuipers J. Experimental and numerical study of wall-induced granular convection [J]. *Powder Technology*, 2008, 184(2): 166-176.
- [23] Zhang F, Wang L, Liu C, Wu P, Zhan S. Patterns of convective flow in a vertically vibrated granular bed [J]. *Physics Letters A*, 2014, 378(18): 1303-1308.
- [24] Sun J, Liu C, Wu P, Xie Z-A, Hu K, Wang L. Granular core phenomenon induced by convection in a vertically vibrated cylindrical container [J]. *Physical Review E*, 2016, 94(3): 032906.
- [25] Evesque P, Rajchenbach J. Instability in a sand heap [J]. *Physical Review Letters*, 1989, 62(1): 44-46.
- [26] Clément E, Duran J, Rajchenbach J. Experimental study of heaping in a two-dimensional "sand pile" [J]. *Physical Review Letters*, 1992, 69(8): 1189-1192.
- [27] King P J, Lopez-Alcaraz P, Pacheco-Martinez H A, Clement C P, Smith A J, Swift M R. Instabilities in vertically vibrated fluid-grain systems [J]. *The European Physical Journal E*, 2007, 22(3): 219-226.
- [28] Rietz F. Segregation and convection rolls in two-dimensional packings [J]. 2013: 771-774.
- [29] Kondic L, Hartley R, Tennakoon S, Painter B, Behringer R. Segregation by friction [J]. *EPL (Europhysics Letters)*, 2003, 61(6): 742.
- [30] Trujillo L, Alam M, Herrmann H J. Segregation in a fluidized binary granular mixture: Competition between buoyancy and geometric forces [J]. *EPL (Europhysics Letters)*, 2003, 64(2): 190.
- [31] Jenkins J T, Yoon D K. Segregation in binary mixtures under gravity [J]. *Phys Rev Lett*, 2002, 88(19): 194301.
- [32] Tarzia M, Fierro A, Nicodemi M, Coniglio A. Segregation in fluidized versus tapped packs [J]. *Phys Rev Lett*, 2004, 93(19): 198002.
- [33] Yuan X, Zheng N, Shi Q, Sun G, Li L. Segregation in mixtures of granular chains and spherical grains under vertical vibration [J]. *Physical review. E, Statistical, nonlinear, and soft matter physics*, 2013, 87(4): 042203.
- [34] Xie Z A, Wu P, Zhang S P, Chen S, Jia C, Liu C P, Wang L. Separation patterns between Brazilian nut and reversed Brazilian nut of a binary granular system [J]. *Physical review. E, Statistical, nonlinear, and soft matter physics*, 2012, 85(6 Pt 1): 061302.
- [35] Vallance J W, Savage S B. Particle segregation in granular flows down chutes. *IUTAM Symposium on Segregation in Granular Flows: Proceedings of the IUTAM Symposium*, A D Rosato, D L Blackmore, Editors, 2000, Springer Netherlands: Dordrecht: 31-51.
- [36] Xie Z-A, Wu P, Su T, Jiang X, Li L, Jia C, Zhang S, Liu C, Wang L. Behaviors of spherical intruder in 3-D vertically vibrating granular system with vertical longitudinal air pressure wave [J]. *Powder Technology*, 2015, 283(Supplement C): 266-285.
- [37] Krengel D, Strobl S, Sack A, Heckel M, Pöschel T. Pattern formation in a horizontally shaken granular submonolayer [J]. *Granular Matter*, 2013, 15(3): 377-387.
- [38] Shimokawa M, Suetsugu Y, Hiroshige R, Hirano T, Sakaguchi H. Pattern formation in a sandpile of ternary granular mixtures [J]. *Physical review. E, Statistical, nonlinear, and soft matter physics*, 1, 91(6): 062205.
- [39] Gray J M N T, Hutter K. Pattern formation in granular avalanches [J]. *Continuum Mechanics and Thermodynamics*, 1997, 9(6): 341-345.

- [40] Ristow G H. Pattern Formation in Granular Matter. Springer Tracts in Modern Physics book series (STMP, volume 164) 2000.
- [41] Makse H A, Havlin S, Ivanov P C, King P R, Prakash S, Eugene Stanley H. Pattern formation in sedimentary rocks: Connectivity, permeability, and spatial correlations [J]. *Physica A: Statistical Mechanics and its Applications*, 1996, 233(3): 587-605.
- [42] Chuan Lim E W. Pattern formation in vibrated beds of dry and wet granular materials [J]. *Physics of Fluids*, 2014, 26(1): 013301.
- [43] Aranson I S, Tsimring L S. Patterns and collective behavior in granular media: Theoretical concepts [J]. *Reviews of modern physics*, 2006, 78(2): 641.
- [44] Orza J A G, Brito R, van Noije T P C, Ernst M H. Patterns and long range correlations in idealized granular flows [J]. *International Journal of Modern Physics C*, 1997, 08(04): 953-965.
- [45] Ansari I H, Alam M. Patterns and velocity field in vertically vibrated granular materials [J]. 2013: 775-778.
- [46] Aoki K M, Akiyama T, Maki Y, Watanabe T. Convective roll patterns in vertically vibrated beds of granules [J]. *Physical Review E*, 1996, 54(1): 874-883.
- [47] Li L, Wu P, Zhang S, Wang L. Diversity and controllability of particle distribution under coupling vibration and airflow [J]. *Soft Matter*, 2017, 13(39): 7034-7045.
- [48] Pihler-Puzovic D, Mullin T. The timescales of granular segregation in horizontally shaken monolayers [C]. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 2013, 469(2157): 20130203-20130203.
- [49] Mullin T. Coarsening of self-organized clusters in binary mixtures of particles [J]. *Physical Review Letters*, 2000, 84(20): 4741-4744.
- [50] Reis P M, Mullin T. Granular segregation as a critical phenomenon [J]. *Physical Review Letters*, 2002, 89(24): 244301.
- [51] Reis P M, Ehrhardt G, Stephenson A, Mullin T. Gases, liquids and crystals in granular segregation [J]. *EPL (Europhysics Letters)*, 2004, 66(3): 357.
- [52] Schnautz T, Brito R, Kruelle C A, Rehberg I. A horizontal Brazil-nut effect and its reverse [J]. *Phys Rev Lett*, 2005, 95(2): 028001.
- [53] Medved M, Dawson D, Jaeger H M, Nagel S R. Convection in horizontally vibrated granular material [J]. *Chaos*, 1999, 9(3): 691-696.
- [54] Liffman K, Metcalfe G, Cleary P. Granular convection and transport due to horizontal shaking [J]. *Physical Review Letters*, 1997, 79(23): 4574.
- [55] Xie Z, Wu P, Wang S, Huang Y, Zhang S, Chen S, Jia C, Liu C, Wang L. Behaviour of a binary particle system under the effects of simultaneous vertical vibration and rotation [J]. *Soft Matter*, 2013, 9(20): 5074.
- [56] Shishodia N, Wassgren C R. Particle segregation in vibrofluidized beds due to buoyant forces [J]. *Physical Review Letters*, 2001, 87(8): 084302.
- [57] Huerta D A, Ruiz-Suárez J C. Vibration-Induced Granular Segregation: A Phenomenon Driven by Three Mechanisms [J]. *Physical Review Letters*, 2004, 92(11): 114301.
- [58] Pereira G G, Tran N, Cleary P W. Segregation of combined size and density varying binary granular mixtures in a slowly rotating tumbler [J]. *Granular Matter*, 2014, 16(5): 711-732.
- [59] Xie Z-A, Wu P, Yang W, Zhao J, Zhang S, Li L, Chen S, Jia C, Liu C, Wang L. Distribution of dissipated energy in a multi-size granular system under vertical vibration [J]. *Powder Technology*, 2014, 260: 1-6.
- [60] Meerson B, Pöschel T, Bromberg Y. Close-packed floating clusters: granular hydrodynamics beyond the freezing point? [J]. *Physical review letters*, 2003, 91(2): 024301.
- [61] Rivas N, Ponce S, Gallet B, Risso D, Soto R, Cordero P, Mujica N. Sudden chain energy transfer events in vibrated granular media [J]. *Physical Review Letters*, 2011, 106(8): 088001.
- [62] Faraday M. On a peculiar class of acoustical figures; and on certain forms assumed by groups of particles upon vibrating elastic surfaces [J]. *Phil.Trans. R. Soc.*, 1831, 131: 299-340.

- [63] Hsiao S S, Chen C H. Granular convection cells in a vertical shaker [J]. Powder Technology, 2000, 111(3): 210-217.
- [64] Taguchi Y h. New origin of a convective motion: Elastically induced convection in granular materials [J]. Physical Review Letters, 1992, 69(9): 1367-1370.
- [65] Hu K, Xie Z-A, Wu P, Sun J, Li L, Jia C, Zhang S, Liu C, Wang L. Convecting particle diffusion in a binary particle system under vertical vibration [J]. Soft Matter, 2014, 10(24): 4348-4359.
- [66] Cafiero R, Luding S, Jürgen Herrmann H. Two-dimensional granular gas of inelastic spheres with multiplicative driving [J]. Physical Review Letters, 2000, 84(26): 6014-6017.
- [67] Tennakoon S, Kondic L, Behringer R. Onset of flow in a horizontally vibrated granular bed: Convection by horizontal shearing [J]. EPL (Europhysics Letters), 1999, 45(4): 470.
- [68] Metcalfe G, Tennakoon S, Kondic L, Schaeffer D, Behringer R. Granular friction, Coulomb failure, and the fluid-solid transition for horizontally shaken granular materials [J]. Physical Review E, 2002, 65(3): 031302.
- [69] Aumaitre S, Puls C, McElwaine J N, Gollub J P. Comparing flow thresholds and dynamics for oscillating and inclined granular layers [J]. Physical review. E, Statistical, nonlinear, and soft matter physics, 2007, 75(6 Pt 1): 061307.
- [70] Poschel T, Rosenkranz D E, Gallas J A. Recurrent inflation and collapse in horizontally shaken granular materials [J]. Physical review. E, Statistical, nonlinear, and soft matter physics, 2012, 85(3 Pt 1): 031307.
- [71] Raihane A, Bonnefoy O, Gelet J L, Chaix J M, Thomas G. Experimental study of a 3D dry granular medium submitted to horizontal shaking [J]. Powder Technology, 2009, 190(1-2): 252-257.
- [72] Nadler S, Bonnefoy O, Chaix J M, Thomas G, Gelet J L. Parametric study of horizontally vibrated grain packings: comparison between Discrete Element Method and experimental results [J]. The European physical journal. E, Soft matter, 2011, 34(7): 66.
- [73] Medved M. Connections between response modes in a horizontally driven granular material [J]. Physical Review E, 2002, 65(2): 021305.
- [74] Laroche C, Douady S, Fauve S. Convective flow of granular masses under vertical vibrations [J]. J. Phys. France, 1989, 50(7): 699-706.
- [75] Gallas J A C, Herrmann H J, Sokołowski S. Convection cells in vibrating granular media [J]. Physical Review Letters, 1992, 69(9): 1371-1374.
- [76] Bizon C, Shattuck M D, de Bruyn J R, Swift J B, McCormick W D, Swinney H L. Convection and Diffusion in Patterns in Oscillated Granular Media [J]. Journal of Statistical Physics, 1998, 93(3): 449-465.
- [77] Ramírez R, Risso D, Cordero P. Thermal convection in fluidized granular systems [J]. Physical Review Letters, 2000, 85(6): 1230-1233.
- [78] Painter B, Behringer R P. Substrate Interactions, Effects of Symmetry Breaking, and Convection in a 2D Horizontally Shaken Granular System [J]. Physical Review Letters, 2000, 85(16): 3396-3399.
- [79] Khain E, Meerson B. Onset of thermal convection in a horizontal layer of granular gas [J]. Physical Review E, 2003, 67(2): 021306.
- [80] Evesque P. Shaking dry powders and grains [J]. Contemporary Physics, 1992, 33(4): 245-261.
- [81] Gallas J, Herrmann H, Sokołowski S. Convection cells in vibrating granular media [J]. Physical review letters, 1992, 69(9): 1371.
- [82] Saluena C, Pöschel T. Convection in horizontally shaken granular material [J]. The European Physical Journal E, 2000, 1(1): 55-59.
- [83] Heckel M, Sack A, Kollmer J E, Pöschel T. Fluidization of a horizontally driven granular monolayer [J]. Physical Review E, 2015, 91(6): 062213.
- [84] Pöschel T, Rosenkranz D E, Gallas J A. Recurrent inflation and collapse in horizontally shaken granular materials [J]. Physical Review E, 2012, 85(3): 031307.
- [85] Rehman A, Wu P, Li L, Zhang S, Wang L. Effects of moment of inertia on energy dissipation and convection motion of

- particles in horizontally vibrated monolayer [J]. Chinese Journal of Physics, 2017:
- [86] Windows-Yule C R K. Convection and segregation in fluidised granular systems exposed to two-dimensional vibration [J]. New Journal of Physics, 2016, 18(3): 033005.
- [87] Windows-Yule C R K, Weinhart T, Parker D J, Thornton A R. Effects of packing density on the segregative behaviors of granular systems [J]. Physical Review Letters, 2014, 112(9): 098001.
- [88] Williams J C. The segregation of particulate materials. A review [J]. Powder Technology, 1976, 15(2): 245-251.
- [89] Rosato A, Strandburg K J, Prinz F, Swendsen R H. Why the Brazil nuts are on top: Size segregation of particulate matter by shaking [J]. Physical Review Letters, 1987, 58(10): 1038-1040.
- [90] Mullin T. Mixing and de-mixing [J]. Science, 2002, 295(5561): 1851.
- [91] Arshad K. Size separation in vibrated granular matter [J]. Reports on Progress in Physics, 2004, 67(3): 209.
- [92] Werner B T, Hallet B. Numerical simulation of self-organized stone stripes [J]. Nature, 1993, 361(6408): 142-145.
- [93] Kessler MA W B. Self-organization of sorted patterned ground [J]. Science, 2003, 299: 380-383.
- [94] Johnson CG K B, Iverson RM, Logan M, LaHusen RG, Gray JMNT. Grain size segregation and levee formation in geophysical mass flows. [J]. J. Geophys. Res., 2012, 117: F01032.
- [95] Clementson C, Ileleji K E, Strohshine R L. Particle segregation within a pile of bulk distillers dried grains with solubles (DDGS) and variability of nutrient content [J]. Cereal Chemistry Journal, 2009, 86(3): 267-273.
- [96] Fan L T, Chen Y-m, Lai F S. Recent developments in solids mixing [J]. Powder Technology, 1990, 61(3): 255-287.
- [97] Engblom N, Saxén H, Zevenhoven R, Nylander H, Enstad G G. Analysis of segregation data for a dry mineral-based construction materials plant [J]. Industrial & Engineering Chemistry Research, 2012, 51(27): 9427-9440.
- [98] Barbosa-Cánovas GV O-R E, Juliano P, Yan H. Food powders: physical properties, processing, and functionality. The Netherlands: Kluwer academic publishers: Dordrecht, 2005 9-53.
- [99] Ahmad K, Smalley I J. Observation of particle segregation in vibrated granular systems [J]. Powder Technology, 1973, 8(1): 69-75.
- [100] Shinbrot T, Muzzio F J. Granular patterns from experiment and model from noise to order [J]. Nature, 2001, 410(6825): 251-258.
- [101] Wiederseiner S, Andreini N, Épely-Chauvin G, Moser G, Monnereau M, Gray J M N T, Ancy C. Experimental investigation into segregating granular flows down chutes [J]. Physics of Fluids, 2011, 23(1): 013301.
- [102] Gray J M N T, Goddard J, Giovine P, Jenkins J T. Particle size segregation in granular avalanches: A brief review of recent progress [J]. AIP Conference Proceedings, 2010, 1227(1): 343-362.
- [103] Seiden G, Thomas P J. Complexity, segregation, and pattern formation in rotating-drum flows [J]. Reviews of Modern Physics, 2011, 83(4): 1323-1365.
- [104] Yi F, Hill K M. Theory for shear-induced segregation of dense granular mixtures [J]. New Journal of Physics, 2011, 13(9): 095009.
- [105] Fan Y, Hill K M. Phase Transitions in Shear-Induced Segregation of Granular Materials [J]. Physical Review Letters, 2011, 106(21): 218301.
- [106] De Silva S R, Dyrøy A, Enstad G G. Segregation Mechanisms and Their quantification using segregation testers. IUTAM Symposium on Segregation in Granular Flows: Proceedings of the IUTAM Symposium, A D Rosato, D L Blackmore, Editors, 2000, Springer Netherlands 11-29.
- [107] Kudrolli A, Wolpert M, Gollub J P. Cluster formation due to collisions in granular material [J]. Physical Review Letters, 1997, 78(7): 1383-1386.
- [108] Ehrhardt G C M A, Stephenson A, Reis P M. Segregation mechanisms in a numerical model of a binary granular mixture [J]. Physical Review E, 2005, 71(4): 041301.
- [109] Windows-Yule C R K, Rivas N, Parker D J. Thermal Convection and Temperature Inhomogeneity in a Vibrofluidized Granular Bed: The Influence of Sidewall Dissipation

- [J]. Physical Review Letters, 2013, 111(3): 038001.
- [110] Windows-Yule C R K, Weinhart T, Parker D J, Thornton A R. Influence of thermal convection on density segregation in a vibrated binary granular system [J]. Physical Review E, 2014, 89(2): 022202.
- [111] Aumaître S, Kruelle C A, Rehberg I. Segregation in granular matter under horizontal swirling excitation [J]. Physical Review E, 2001, 64(4): 041305.
- [112] Ciamarra M P, Coniglio A, Nicodemi M. Shear Instabilities in granular mixtures [J]. Physical Review Letters, 2005, 94(18): 188001.
- [113] Pooley C M, Yeomans J M. Stripe formation in differentially forced binary systems [J]. Physical Review Letters, 2004, 93(11): 118001.
- [114] Fujii M, Awazu A, Nishimori H. Segregation-pattern reorientation of a granular mixture on a horizontally oscillating tray [J]. Physical Review E, 2012, 85(4): 041304.
- [115] Ciamarra M P, Coniglio A, Nicodemi M. Dynamically induced effective interaction in periodically driven granular mixtures [J]. Physical Review Letters, 2006, 97(3): 038001.
- [116] Reis P M, Sykes T, Mullin T. Phases of granular segregation in a binary mixture [J]. Physical Review E, 2006, 74(5): 051306.
- [117] Strassburger G, Betat, A., Scherer, M.A., Rehberg, I. Pattern formation by horizontal vibration of granular material [J]. Workshop on Traffic and Granular Flow, 1995, World Scientific: 329–334.
- [118] Sánchez P, Swift M R, King P J. Stripe Formation in Granular Mixtures due to the Differential Influence of Drag [J]. Physical Review Letters, 2004, 93(18): 184302.
- [119] Robabeh M, Maniya M, Shaebani M R, Ruiz-Suárez J C, Eric C. Stripe formation in horizontally oscillating granular suspensions [J]. EPL (Europhysics Letters), 2014, 107(3): 34006.