# Comparative Studies on Techniques for Transportation Problem of Initial Basic Feasible Solution (IBFS) 

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#### Abstract

Transportation model is a special class of linear programming problem in which the objective is to transport or distribute a single commodity or goods from various sources/origins to different destination at a maximum total cost. There are three methods to solve initial basic feasible solution in transportation problem (IBFS) all these methods were consider and their results were comparing with each other's. The research finding shows that Vogel's Approximation method (operation method) is better than all other methods such as Least cost method (Business man's method) and Northwest corner method (layman's method).i.e. VAM<LCM<NWCM ( $\mathrm{N}=3520<\mathrm{N} 3670<\mathrm{N} 3680$ ). This study will enable companies to take advantage of this opportunity to improve on their supply chain. Based on the study findings as presented in (VAM), the study recommends that optimum solution can be attained through the following routes: $I B, I D$, $O C, O D, O A$, and $E A$.


Keywords: Transportation problem; balanced transportation problem; unbalanced transportation problem; dummy; origin; destination; northwest corner method; least cost method; Vogel's Approximation method and Shipment.

## 1. Introduction

Transportation model is a special class of linear programming problem in which the objective is to transport or distribute a single commodity or goods from various sources/origins to different destination at a maximum total cost.
Transportation model is a model for capacity planning and scheduling. As a tool, transportation problem (TP) is a specialized form of linear programming problem. This problem involves M sources, each of which has available a $\iota(\iota=$ $1,2,3 \ldots m$ ) units of a homogeneous product, and $N$ destinations, each of which requires $b_{j}(\mathfrak{j}=1,2$,
$3 \ldots \mathrm{n}$ ) units of this products. The number $\mathrm{a} \ell$ and bj are positive integers (Abara, 2011).
Transportation problems arise in all such cases. It aims at providing assistance to top management in ascertaining how many units of a particular product should be transported from plant to each depot to that the total prevailing demand for the company's product is satisfied, while at the same time the total transportation costs are minimized (Hamdy, 2008).
Lee et al (1981) argue that transportation problem deals with the transportation of a product from a number of sources with limited supplies, to a
number of destinations, with specific demands, at the minimum total transportation cost.
There are two types of transportation problem namely:

- Balanced transportation problem
- Unbalanced transportation problem
1.1 Definition of balanced transportation problem: A transportation problem is said to be balanced transportation problem if total number of supply is same as total number of demand that is

$$
\begin{equation*}
\sum_{i=1}^{m} a i=\sum_{j=1}^{n} b j \tag{1}
\end{equation*}
$$

### 1.2 Definition of unbalanced transportation

 problem: Transportation is said to be unbalanced transportation problem if total number of supply is not same as total number of demandi.e $\sum_{i=1}^{m} a i \neq \sum_{j=1}^{n} b j$
1.3 Dummy Origin/Destination: A dummy origin or destination is an imaginary origin or destination with zero cost introduced to make an unbalanced transportation problem balanced. If the total supply is more than the total demand we introduce an additional column which will indicate the surplus supply with transportation
cost zero. Likewise, if the total demand is more than the total supply, an additional row is introduced in the Table, which represents unsatisfied demand with transportation cost zero
1.4 Initial Basic Feasible Solution: If solution $X \imath \geq 0$ is said to be a feasible solution of a transportation problem if it satisfies, the constraints. The feasible solution is said to be basic feasible solution if the following condition are satisfy

- The problem must be balanced
- The number of cell allocation must be equal to $\mathrm{M}+\mathrm{N}-1$, where $\mathrm{M} \& \mathrm{~N}$ are numbers of rows and columns respectively.
Any solution satisfying the two conditions is termed Non- Degenerate Basic Feasible Solution (NDBFS) otherwise it is called Degenerate solution (DS)


### 1.5 Formulation of a general Transportation

 Algorithm. The table below represents a typical transportation problem with M number of sources and N number of destination. The transportation cost from $\mathrm{i}^{\text {th }}$ source to $\mathrm{j}^{\text {th }}$ destination is $\mathrm{C}_{\mathrm{ij}}$ and the amount shipped is $\mathrm{X} \iota \jmath$Table 1: Transportation Algorithm

| Destination/sources | D1 | D2 | D3 | N | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| S1 | $\mathrm{a}_{11}$ | $\mathrm{a}_{12}$ | $\mathrm{a}_{13}$ | $\mathrm{a}_{1 \mathrm{n}}$ | $\mathrm{S}_{1}$ |
| S2 | $\mathrm{a}_{21}$ | $\mathrm{a}_{22}$ | $\mathrm{a}_{23}$ | $\mathrm{a}_{2 \mathrm{n}}$ | $\mathrm{s}_{2}$ |
| S3 | $\mathrm{a}_{31}$ | $\mathrm{a}_{32}$ | $\mathrm{a}_{33}$ | $\mathrm{a}_{3 \mathrm{n}}$ | $\mathrm{s}_{3}$ |
| M | $\mathrm{a}_{\mathrm{m} 1}$ | $\mathrm{a}_{\mathrm{m} 2}$ | $\mathrm{a}_{\mathrm{m} 3}$ | $\mathrm{a}_{\mathrm{mn}}$ | $\mathrm{s}_{\mathrm{m}}$ |
| Demand | $\mathrm{d}_{1}$ | $\mathrm{d}_{2}$ | $\mathrm{d}_{3}$ | $\mathrm{d}_{\mathrm{n}}$ | $\sum s i=\sum d j$ |

The standard mathematical model for the problem is
Minimize $\mathrm{Z}=\sum_{i=1}^{m} \sum_{j=1}^{n} C i j X i j$
Subject to $\quad \sum_{j=1}^{n} X i j=S i(i=1,2,3, \ldots, m)$

$$
\begin{equation*}
\sum_{i=1}^{m} X i j=d i(i=1,2,3, \ldots, n) \quad 5 \tag{4}
\end{equation*}
$$

With all $\mathrm{X} \ell \jmath$ non negative and integral

## 2. Objective of the study

The objective of this study is to determine the best methods of minimizing transportation cost using the initial basic feasible solution models of transportation listed below

- Northwest corner method (layman's method)
- Least cost method (Business man's method)
- Vogel's Approximation method (operation method)


## 3. Methodology

This section provides discussion on the initial Basic Feasible Solution Method of solving transportation problem. All the three method of solving initial basic feasible solution will be used and their results will be comparing with each other.

### 3.1 North West Corner Method (NWCM)

The method is the simplest but most inefficient as it has the highest total transportation cost is comparison to all other methods. The main reason that can be attributed to this is that the method does not take into account the cost of transportation for all the possible alternative routes.
The practical steps involved in the North-West Comer Method are given below:
Step 1: Make maximum possible allocation to the Upper-Left Comer Cell (also known as NorthWest Comer Cell) in the First Row depending upon the availability of supply for that Row and demand requirement for the Column containing that Cell. Note: Unit transportation cost is completely ignored. Step 2: Move to the Next Cell of the First Row depending upon remaining supply for that Row and the demand requirement for the next Column. Go on till the Row total is exhausted.
Step 3: Move to the next Row and make allocation to the Cell below the Cell of the preceding Row in which the last allocation was made and follow Steps I and 2.
Step 4: Follow Steps I to 3 till all Rim requirements are exhausted, i.e., the entire demand and supply is exhausted

### 3.2 Lest Cost Method (LCM)

In this method the cheapest route is always the focus for allocation. It is a better method compared to NCWR because costs are considered for allocation.

The practical steps involved in the Least Cost Method are given below:
Step 1: Make maximum possible Allocation to the Least. Cost Cell depending upon the demand/supply for the Column Row containing that Cell. In case of Tie in the Least Cost Cells, make allocation to the Cell by which maximum demand or capacity is exhausted.
Step 2: Make allocation to the Second Lowest Cost Cell depending upon the remaining demand/supply for the Row/ Column containing that Cell.
Step 3: Repeat the above Steps till all Row Requirements are exhausted, i.e., entire demand and supply is exhausted

### 3.3 Vogel's Approximations Method (VAM)

VAM, which is also called penalty method, is an improvement on the LCM method that generates a better initial solutions; it makes use of opportunity cost (penalty) principles in order to make allocation to various cells by minimizing the penalty cost. The steps in this method are:
The practical steps involved in Vogel's Approximation Method (or VAM) are given below:
Step 1: Row Difference: Find the difference between Smallest and Second Smallest element of each Row, representing the. Opportunity Cost of not making the allocation to the Smallest Element Cell, and write the difference on the right-hand side of the concerned Row. In case of tie between two smallest elements, the difference should be taken as zero.
Step 2: Column Difference: Find the difference between Smallest and Second Smallest element of each column, representing the Opportunity Cost of not making the allocation to the Smallest Element Cell, and write the difference below the concerned Column. In case of tie between two smallest elements, the difference should be taken as zero.
Step 3: Make the Largest Difference amongst all Differences by an arrow indicating the allocation to be made to the row/ column having largest difference. Allocate maximum possible quantity to the Least Cost Cell of the selected row/column
depending upon the quantity available. In case of tie between the Differences, select the row or column having least cost cell. However, in case of tie even in case of Least Cost, make allocation to that Cell by which maximum requirements are exhausted.
Step 4: Shade the Row/Column whose availability or requirement is exhausted so that it shall not be considered for any further allocation.
Step 5: Repeat Step 3 and 4 till entire demand and supply is exhausted.
Step 6: Draw the Initial Feasible Solution Table obtained after the above steps.

## 4. Presentation and Analysis

In this section, the data on transportation problem for this paper we shall adopted numerical illustration to compare the results of (IBFS)

## Illustration:

XYZ ventures limited has three furniture workshops located at I, O and E with production capacities of 120,70 and 50 sets per week respectively. These are to be shipped to four depots at $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D with requirements of 60 , 40,30 and 110 sets respectively. The transportation costs in naira per set between factories and depots are given in the table below.

Table 2: North West Corner Method (NWCM)

|  | A | B | C | D | Supply |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| I | 20 | $[60]$ | 22 | $[40]$ | 17 | $[\underline{20]}$ | 4 |  |
| O | 24 |  | 37 |  | 9 | $[10]$ | 7 | $[60]$ |
| 70 | 70 | 60 |  |  |  |  |  |  |
| E | 32 | 37 |  | 20 |  | 15 | $[50]$ | 50 |
| Demand | 60 |  | 40 |  | 30 | 10 | 710 | 50 |

## Check for balance:

Total Demand $=60+40+30+110=240$
Total Supply $=120+70+50=240$
Since total demand = total supply (no degeneracy)
Shipment
IA $=20 * 60=120$

IB $=22 * 40=880$
IC $=17 * 20=340$
$\mathrm{OC}=9 * 10=90$
$\mathrm{OD}=7 * 60=420$
$\mathrm{ED}=15 * 50=750$
Total cost $=\mathrm{N} \mathbf{3 6 8 0}$

Table 3: Lest Cost Method (LCM)

|  | A |  | B |  | C |  | D | supply |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 20 | [10] | 22 |  | 17 |  | 4 | [110] | 120 | T0 |
| O | 24 | [40] | 37 |  | 9 | [30] | 7 |  | 79 | 40 |
| E | 32 | [10] | 37 | [40] | 20 |  | 15 |  | 50 | 40 |
| Demand | 60 | T0 | 40 |  | 30 |  | 710 |  |  |  |

## Check for balance:

Total Demand $=60+40+30+110=240$
Total Supply $=120+70+50=240$
Since total demand = total supply (no degeneracy)

## Shipment

IA $=20 * 10=200$
ID $=4 * 110=440$
$\mathrm{OA}=24 * 40=960$
$\mathrm{EC}=9 * 30=270$
$\mathrm{OA}=32 * 10=320$
$\mathrm{EB}=37 * 40=1480$
Total Cost $=\mathrm{N} \mathbf{3 6 7 0}$

Table 4: Vogel Approximation Method
Tableau I

|  | A | B | C | D | supply | penalty |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| I | 20 | 22 | $[40]$ | 17 | 4 | 420 | 80 |
| O | 24 | 37 |  | 9 | 7 | 70 | 2 |
| E | 32 | 37 |  | 20 | 15 | 50 | 5 |
| Demand | 60 | 40 | 30 | 110 |  |  |  |
| penalty | 4 | 15 | 8 | 3 |  |  |  |

Check for balance: $\quad$ Total Supply $=120+70+50=240$
Total Demand $=60+40+30+110=240 \quad$ Since total demand $=$ total supply (no degeneracy)
Tableau II

|  | A | C | D | supply | penalty |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| I | 20 | 17 | 4 | [80] | 80 | 13 |
| O | 24 | 9 | 7 |  | 70 | 2 |
| E | 32 | 20 | 15 |  | 50 | 5 |
| Demand | 60 | 30 | 710 | 30 |  |  |
| penalty | 4 | 8 | 3 |  |  |  |

Tableau III

|  | A | C | D | supply | penalty |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| O | 24 | 9 | $[30]$ | 7 | 70 | 40 | 2 |
| E | 32 | 20 |  | 15 | 50 | 5 |  |
| Demand | 60 | 30 |  | 30 |  |  |  |
| Penalty | 8 | 11 |  | 8 |  |  |  |

Tableau IV

|  | A | D | Supply |  | Penalty |
| :--- | :--- | :--- | :--- | :--- | :--- |
| O | 24 | 7 | $[30]$ | 40 | 10 |
| 17 |  |  |  |  |  |
| E | 32 | 15 | 50 |  | 17 |
| Demand | 60 | 30 |  |  |  |
| Penalty | 8 | 8 |  |  |  |

## Tableau V

|  | A | Supply |  |
| :--- | :--- | :--- | :--- |
| D | $24[10]$ | 40 |  |
| E | 32 | $530]$ | 50 |
| Demand | 60 |  |  |
| Penalty | 8 |  |  |

## Shipment

IB $=22 * 40=880$
ID $=4 * 80=320$
OC $=9 * 30=270$
$\mathrm{OD}=7 * 30=210$
$\mathrm{OA}=24 * 10=240$
$\mathrm{EA}=32 * 50=1600$
Total Cost $=\mathrm{\#} 3520$

## 5. Comparison

Comparison of total cost of Transportation Problem of above examples between (IBFS)

- Northwest corner method (layman's method) is $\mathrm{N} \mathbf{3 6 8 0}$
- Least cost method (Business man's method) is $\# \mathbf{3 6 7 0}$
- Vogel's Approximation method (operation method) is $\mathbf{N 5 2 0}$


## 6. Conclusion

In this Paper, we had discoveredthatVogel's Approximation method (operation method) is better than all other methods such as Least cost method (Business man's method) and Northwest corner method (layman's method).i.e. VAM $<L C M<N W C M \quad(N-3520<$ N $3670<$ N 3680). In this paper we can also advice manager of XYZ ventures limited to make use of Vogel's Approximation method (operation method) as theirbest mode of (IBFS) in transportation problem. Based on the study findings as presented in VAM, the study recommends that optimum solution can be attained through the following routes:IB, ID, OC, OD, OA, and EA

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