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## A study of production inventory model of deteriorating items in an imperfect production process

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**Abstract:**

The paper describes an EMQ (Economic Manufacturing Quantity ) model for a deteriorating item considering time-dependent demand. The deterioration rate is a constant fraction of the on hand inventory and we have considered three types of demand like, quadratic, exponential and stock dependent. Here our objective is to determine optimal product reliability and production rate that achieves the maximum profit for an imperfect manufacturing process. We use the Euler-Lagrangin method to find the optimal solution for product reliability parameter and dynamic production rate. It is then illustrated with the help of numerical examples.

**Keywords:** Production Inventory model, Deterioration, Product reliability, Imperfect production.

**1. Introduction:**

Formulation of inventory models depends on two important factors , deterioration of items and variation in the demand rate with time. Deterioration of items should not be neglected for all items. Some items like foodstuff, pharmaceuticals, chemicals, blood, fish, radioactive chemicals and food grains etc. undergo deterioration. Many models for dealing with deteriorating items have been proposed over the past several decades. Researchers like Aggarwal (1978), Dave and Patel (1981), Roychaudhuri and Chaudhuri (1983), Dave (1979), Bahari-Kashani (1989), Wee(1995), Hariga (1996a, 1996b), Benkherouf (1995,1997,1998) etc., assumed that items deteriorate at a constant rate. Covert and Philip (1973) used a two parameter weibull distribution to represent the distribution of time to deterioration. Philip (1974) generalised the model by taking three parameter weibull distribution deterioration. Further, Deb and Chaudhuri (1986), Giri, Goswami and Chaudhuri (1996), Ghosh and Chaudhuri (2006), Sana (2010a, 2010b) etc. ,considered time-proportional deterioration.

Many inventory models have been developed considering linear, exponential and quadratic time dependent demand. Some goods are there whose demands continue to be indefinitely large for long time. For example, spare parts of new aero-planes, computer chips of advanced computer machines, mobile phones, newly launched fashionable products in super market etc.. Here demand can be represented by an exponential function of time. Some inventory models developed in this region by Goswami and Chaudhuri (1991) and (1992), Chakraborty and Chaudhuri (1997), Giri and Chaudhuri (1997) etc. On the other hand some inventory models have been developed considering quadratic time dependent demand rate. This type of demand used in the models of Khanra and Chaudhuri (2003), Ghosh and Chaudhuri (2006), Manna, Chaudhuri and Chang (2007) etc.. Some modelers developed their inventory models consider a stock

dependent demand. The works in this area have been by Shah and Jaiswal (1977), Urban (1995), Reddy and Sarma (2001), Sana and Chaudhuri (2004), Zhou and Yang (2004) etc.

Many researchers have extensively discussed various types of inventory models considering perfect and imperfect quality products. All manufacturing items that produced in a production system are, not all of perfect quality. The imperfect quality items are processed to make it perfect, and for that a cost is required. This cost, known as rework cost depends product reliability parameter and production rate. Chang (2004) investigated the inventory problem for imperfect quality. There he proposed two fuzzy models, considering defective production rate and annual demand. Sana et. al(2003, 2004) developed a volume flexible inventory model in an imperfect production process. Considering imperfect production system, an economic production lot size model developed by Das Roy and Sana, S. (2011), which is the extension of the model of Khouja and Mehrez (1994).

In our paper, we have extended the model of Deb Roy (2011) by considering a production inventory model over a finite planning horizon in an imperfect production process. The demand rate and production rate varies with time. We consider three types of demand like quadratic, exponential and stock dependent demand. The inventory is assumed to deteriorate at a constant rate and shortages are not allowed here. The solution procedure of the model is illustrated with the help of numerical examples.

## 2. Assumptions and Notations:

The following assumptions have been used in developing the model:

1. This model is developed only for a single item in an imperfect production process. In this process, two types of goods are produced; one is perfect quality and other is imperfect quality (defective).
2. A cost is charged for rework/disposal of the defective items.
3. The deterioration at time  $t$  is considered as a constant.
4. The initial and terminal inventory level is zero.
5. Unit production cost is a function of product reliability parameter and production rate is  $p(t)$ .
6. Demand at time  $t$  is considered as the increasing function of time.
7. Time horizon is finite.

The inventory model is developed considering the following notations.

1.  $Q(t)$ : on-hand inventory at time  $t > 0$ .
2.  $\dot{Q}(t)$ : derivative of  $Q(t)$  with respect to time  $t$ .
3.  $f(t)$ : demand at time  $t > 0$ .
4.  $p(t)$ : production rate at time  $t$ .
5.  $\theta$ : product reliability parameter.
6.  $\theta_{min}$ : Minimum value of  $\theta$ .
7.  $\theta_{max}$ : Maximum value of  $\theta$ .
8.  $V(\theta)$ : development cost for production system.
9.  $A$ : The fixed cost like labor and energy costs which is independent of reliability parameter  $\theta$ .
10.  $B$ : The cost of technology, resource and design complexity for production when  $\theta = \theta_{max}$ .
11.  $K$ : Represents the difficulties in increasing reliability, which depends on the design complexity, technological and resource limitations, etc.
12.  $C_r$ : Material cost per unit item.
13.  $\alpha$ : Variation constant of tool/die costs.
14.  $C(\theta; t)$ : Production cost of unit item.
15.  $W$ : selling price per unit item solType equation here.d.
16.  $C_h$ : Inventory holding cost per unit per unit time.
17.  $R$ : rework/disposal cost per defective item.

- 18.  $\delta$ : r-i, where r is the interest per unit currency and i is the inflation per unit currency.
- 19. L: the length of production {inventory cycle}.

**3. Formulation of the model**

Most of the production system produces perfect and imperfect quality items. Here  $\theta$  represents product reliability parameter. The smaller value of provides better quality product. In this paper we refer to the model presented by Shib Sankar Sana (2009), in which the development cost for production system is modeled as

$$V(\theta) = A + B e^{k(\theta_{max}-\theta)/(\theta-\theta_{min})} \tag{1}$$

Where  $\theta$  is the product reliability parameter. Here, A is the fixed cost like labor and energy costs which is independent of reliability factor  $\theta$ , B is the cost of technology, resource and design complexity for production when  $\theta=\theta_{max}$ . The constant K represents the difficulties in increasing reliability, which depends on design complexity, technological and resource limitations etc. improves resource coordination should translate into a greater ability to satisfy demand with customized responses by making better use of inventory. When  $\theta$  dType equation here.ecreases the product becomes more reliable, the development cost increases. We assume that the value of  $\theta$  lies in the interval  $[\theta_{min}, \theta_{max}]$ . We consider the unit production cost as,

$$C(\theta, t) = C_r + \frac{V(\theta)}{p(t)} + \alpha p(t) \tag{2}$$

Where  $C_r$  is the material cost per unit which is fixed,  $\frac{V(\theta)}{p(t)}$  is the development cost which is equally distributed over the production p (t) at time t,  $\alpha p(t)$  is tool/die cost which is proportional to the production rate.

Here, C ( $\theta$ , t) is minimum at  $p(t)=\sqrt{\frac{V(\theta)}{\alpha}}$  . At time t=0, the inventory cycle starts with zero inventory and ends with zero inventory at time t=L. The production level and inventory during time span [0, L] is given as,

$$\frac{dQ(t)}{dt} + \gamma Q(t) = p(t) - f(t) \tag{3}$$

with  $Q(0) = 0$  and  $Q(L) = 0$ . Here,  $\gamma$  is the deterioration rate which is constant over time. The total profit considering deterioration, inflation and time value of money is

$$\pi = \int_0^L e^{-\delta t} \{ (W - C(\theta, p(t)))p(t) - C_h Q - R\theta p(t) - \gamma W Q \} dt \tag{4}$$

Therefore, using equation (2) and equation (3), the profit function, without inflation becomes

$$\begin{aligned} \pi &= \int_0^L e^{-\delta t} \left\{ \left( W - C_r - \frac{V(\theta)}{p(t)} - \alpha p(t) \right) p(t) - C_h Q - R\theta p(t) - \gamma W Q \right\} dt \\ &= \int_0^L e^{-\delta t} \{ (W - C_r - R\theta)p(t) - C_h Q - V(\theta) - \alpha(p(t))^2 - \gamma W Q \} dt \end{aligned}$$

$$\begin{aligned}
&= \int_0^L e^{-\delta t} \{ (W - C_r - R\theta)(\dot{Q} + \gamma Q + f) - C_h Q - V(\theta) - \alpha(\dot{Q} + \gamma Q + f)^2 - \gamma W Q \} dt \\
&= \int_0^L \phi(\dot{Q}, Q, t) dt \tag{5}
\end{aligned}$$

Where,

$$\begin{aligned}
\phi(\dot{Q}, Q, t) &= e^{-\delta t} \{ (W - C_r - R\theta)(\dot{Q} + \gamma Q + f) - C_h Q - V(\theta) - \alpha(\dot{Q} + \gamma Q + f)^2 - \gamma W Q \} \\
&= e^{-\delta t} \{ -\alpha \dot{Q}^2 + (W - C_r - R\theta)\dot{Q} - 2\alpha\gamma Q \dot{Q} - 2\alpha f \dot{Q} - (C_r\gamma + R\theta\gamma + 2\alpha\gamma f + C_h)Q \\
&\quad - \alpha\gamma^2 Q^2 + (W - C_r - R\theta)f - V(\theta) - \alpha f^2 \} \tag{6}
\end{aligned}$$

Here, our objective is to find the optimal value of Q (t) and p (t) such that  $\pi$  is maximized.

The Euler-Lagrange's equation for the maximum value of  $\pi$  is

$$\frac{d}{dt} \left( \frac{\partial \phi}{\partial \dot{Q}} \right) - \frac{\partial \phi}{\partial Q} = 0 \tag{7}$$

Where,

$$\begin{aligned}
\frac{\partial \phi}{\partial \dot{Q}} &= e^{-\delta t} \{ -2\alpha \dot{Q} + (W - C_r - R\theta) - 2\alpha\gamma Q - 2\alpha f \} \\
\frac{\partial \phi}{\partial Q} &= e^{-\delta t} \{ -2\alpha\gamma \dot{Q} - (C_r\gamma + R\theta\gamma + 2\alpha\gamma f + C_h) - 2\alpha\gamma^2 Q \}
\end{aligned}$$

From equation (7) we have

$$\ddot{Q} - \delta Q - (\gamma^2 + \gamma\delta)Q = -\omega(t) \tag{8}$$

$$\text{Where, } \omega(t) = \dot{f} - \delta f + \frac{1}{2\alpha}(W - C_r - R\theta)\delta - \frac{1}{2\alpha}(C_r\gamma + R\theta\gamma + 2\alpha\gamma f + C_h)$$

The solution of the equation (8) is given by

$$Q(t) = X e^{-\gamma t} + Y e^{(\gamma+\delta)t} + \frac{1}{[D^2 - \delta D - (\gamma^2 + \delta\gamma)]} (-\omega(t)) \tag{9}$$

where, X and Y are constant and  $-\omega(t)$  is given by

$$-\omega(t) = (\delta + \gamma)f - f + \frac{1}{2\alpha}(C_h - WQ) + \frac{(\delta+\gamma)}{2\alpha}(C_r + R\theta) \tag{10}$$

Case1: When the demand is quadratic function of time and of the form  $f(t) = a + bt + ct^2$ ,  
 $b, c > 0, a \neq 0$ .

From equation (9) we get

$$Q(t) = X e^{-\gamma t} + Y e^{(\gamma+\delta)t} - \frac{1}{\gamma}(a + bt + ct^2) - \frac{2c}{\gamma^3} + \frac{b+2ct}{\gamma^2} + \frac{W\delta - C_h}{2\alpha(\gamma^2 + \delta\gamma)} - \frac{1}{2\alpha\gamma}(C_r + R\theta) \tag{11}$$

Using the condition  $Q(0) = 0$  and  $Q(L) = 0$  we have

$$X + Y = \frac{a\gamma^2 - b\gamma + 2c}{\gamma^3} - \frac{W\delta - C_h}{2\alpha(\gamma^2 + \delta\gamma)} + \frac{1}{2\alpha\gamma} (C_r + R\theta) \quad (12)$$

and

$$Xe^{-\gamma L} + Ye^{(\gamma + \delta)L} = \frac{a\gamma^2 + bL\gamma^2 + cL^2\gamma^2 + 2c - b\gamma - 2cL\gamma}{\gamma^3} - \frac{W\delta - C_h}{2\alpha(\gamma^2 + \delta\gamma)} + \frac{1}{2\alpha\gamma} (C_r + R\theta) \quad (13)$$

Solving equation (12) and (13) we have,

$$Y = \frac{1}{(e^{-\gamma L} - e^{(\gamma + \delta)L})} \left[ \frac{a\gamma^2 - b\gamma + 2c}{\gamma^3} (e^{-\gamma L} - 1) - \frac{bL\gamma^2 + cL^2\gamma^2 - 2cL\gamma}{\gamma^3} - \frac{W\delta - C_h}{2\alpha(\gamma^2 + \delta\gamma)} (e^{-\gamma L} - 1) + \frac{(C_r + R\theta)}{2\alpha\gamma} (e^{-\gamma L} - 1) \right] \quad (14)$$

And from this value of Y the value of X can be obtained from equation (12) as

$$X = \frac{a\gamma^2 - b\gamma + 2c}{\gamma^3} - \frac{W\delta - C_h}{2\alpha(\gamma^2 + \delta\gamma)} + \frac{(C_r + R\theta)}{2\alpha\gamma} - Y \quad (15)$$

Now from equation (11), we will get the value of Q (t) and the value of p(t) is obtained from equation (3). The profit function ( ) can be obtained from equation (5). This profit function  $\pi(\theta)$  has maximum value if  $\frac{d\pi}{d\theta} = 0$  and  $\frac{d^2\pi}{d\theta^2} < 0$ .

Case 2: when the demand is exponential function of time and of the form  $f(t) = ae^{bt}$ ,  $a, b > 0$ .

From equation (9), we get

$$Q(t) = Xe^{-\gamma t} + Ye^{(\gamma + \delta)t} + \frac{(\delta + \gamma)(ae^{bt})}{(b^2 - b\delta - \gamma^2 - \gamma\delta)} - \frac{abe^{bt}}{(b^2 - b\delta - \gamma^2 - \gamma\delta)} + \frac{W\delta - C_h}{2\alpha(\gamma^2 + \gamma\delta)} - \frac{1}{2\alpha\gamma} (C_r + R\theta) \quad (16)$$

Using the condition Q (0) = 0 and Q (L) = 0 from equation (16), we get,

$$X + Y = \frac{1}{2\alpha\gamma} (C_r + R\theta) + \frac{a}{b + \gamma} - \frac{W\delta - C_h}{2\alpha(\gamma^2 + \gamma\delta)} \quad (17)$$

and

$$Xe^{-\gamma L} + Ye^{(\gamma + \delta)L} = \frac{1}{2\alpha\gamma} (C_r + R\theta) + \frac{ae^{bL}}{(b + \gamma)} - \frac{W\delta - C_h}{2\alpha(\gamma^2 + \gamma\delta)} \quad (18)$$

Solving equation (17) and equation (18), we get,

$$Y = \frac{1}{(e^{-\gamma L} - e^{(\gamma + \delta)L})} \left[ \frac{(C_r + R\theta)}{2\alpha\gamma} (e^{-\gamma L} - 1) - \frac{W\delta - C_h}{2\alpha(\gamma^2 + \gamma\delta)} (e^{-\gamma L} - 1) - \frac{W\delta - C_h}{2\alpha(\gamma^2 + \gamma\delta)} (e^{-\gamma L} - 1) + \frac{a(e^{-\gamma L} - e^{bL})}{b + \gamma} \right] \quad (19)$$

And from this value of Y the value of X can be obtained from equation (17) as

$$X = \frac{1}{2\alpha\gamma} (C_r + R\theta) - \frac{W\delta - C_h}{2\alpha(\gamma^2 + \gamma\delta)} + \frac{a}{b + \gamma} - Y \quad (20)$$

Now from equation (16), we will get the value of  $Q(t)$  and the value of  $p(t)$  is obtained from equation (3). The profit function  $\pi(\theta)$  can be obtained from equation (5). This profit function has maximum value if  $\frac{d\pi}{d\theta} = 0$  and  $\frac{d^2\pi}{d\theta^2} < 0$ .

Case 3: When the demand is dependent on the level of inventory and of the form  $f(t) = a + bQ(t)$ ,  $a, b > 0$ .

Then, the profit function  $\pi$  is given by

$$\begin{aligned} \pi &= \int_0^L e^{-\delta t} \{ (W - C_r - R\theta)(\dot{Q} + \gamma Q + f) - V(\theta) - \alpha(\dot{Q} + \gamma Q + f)^2 - (C_h + \gamma W)Q \} dt \\ &= \int_0^L e^{-\delta t} [-\alpha\dot{Q}^2 + \{ (W - C_r - R\theta) - 2\alpha a \} Q - 2\alpha a(b + \gamma)Q\dot{Q} \\ &\quad + \{ (b + \gamma)(W - C_r - R\theta) - C_h - \gamma W - 2\alpha a(b + \gamma) \} Q - \alpha(b + \gamma)^2 Q^2 \\ &\quad - \alpha a^2 - V(\theta) + a(W - C_r - R\theta)] dt \\ &= \int_0^L \phi(\dot{Q}, Q, t) dt \end{aligned} \tag{21}$$

Where,

$$\begin{aligned} \phi(\dot{Q}, Q, t) &= e^{-\delta t} [-\alpha\dot{Q}^2 + \{ (W - C_r - R\theta) - 2\alpha a \} Q - 2\alpha a(b + \gamma)Q\dot{Q} \\ &\quad + \{ (b + \gamma)(W - C_r - R\theta) - C_h - \gamma W - 2\alpha a(b + \gamma) \} Q - \alpha(b + \gamma)^2 Q^2 \\ &\quad - \alpha a^2 - V(\theta) + a(W - C_r - R\theta)] \end{aligned} \tag{22}$$

Now, the Euler-Lagrange's equation for the maximum value of  $\pi(\theta)$  is

$$\frac{d}{dt} \left( \frac{\partial \phi}{\partial \dot{Q}} \right) - \frac{\partial \phi}{\partial Q} = 0 \tag{23}$$

Using equation (22), equation (23) reduces to

$$\ddot{Q} - \delta Q - (b + \gamma)(b + \delta + \gamma)Q = \omega(t) \tag{24}$$

where,

$$\omega(t) = a\delta - \frac{1}{2\alpha}(W - C_r - R\theta)(b + \delta + \gamma) + \frac{1}{2\alpha}(C_h + \gamma W) + a(b + \gamma) \tag{25}$$

The solution of equation (24) is given by

$$Q(t) = X e^{(b+\delta+\gamma)t} + Y e^{-(b+\gamma)t} - \frac{a}{b+\gamma} + \frac{(W-C_r-R\theta)}{2\alpha(b+\gamma)} - \frac{C_h+\gamma W}{2\alpha(b+\gamma)(b+\gamma+\delta)} \tag{26}$$

Using the condition  $Q(0) = 0$  and  $Q(L) = 0$  we get,

$$X + Y = \frac{a}{b+\gamma} - \frac{(W-C_r-R\theta)}{2\alpha(b+\gamma)} + \frac{C_h+\gamma W}{2\alpha(b+\gamma)(b+\gamma+\delta)} \quad (27)$$

and

$$Xe^{(b+\delta+\gamma)L} + Ye^{(b+\gamma)L} = \frac{a}{b+\gamma} - \frac{(W-C_r-R\theta)}{2\alpha(b+\gamma)} + \frac{C_h+\gamma W}{2\alpha(b+\gamma)(b+\gamma+\delta)} \quad (28)$$

Solving equation (27) and equation (28), we get,

$$Y = \frac{1}{e^{(b+\delta+\gamma)L} - e^{-(b+\gamma)L}} \left[ e^{(b+\delta+\gamma)L} - 1 \right] \left[ \frac{a}{b+\gamma} - \frac{(W-C_r-R\theta)}{2\alpha(b+\gamma)} + \frac{C_h+\gamma W}{2\alpha(b+\gamma)(b+\gamma+\delta)} \right] \quad (29)$$

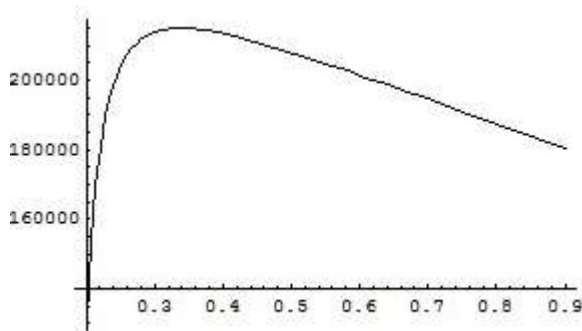
From this value of Y, the value of X can be obtained from equation (27) as

$$X = \frac{a}{b+\gamma} - \frac{(W-C_r-R\theta)}{2\alpha(b+\gamma)} + \frac{C_h+\gamma W}{2\alpha(b+\gamma)(b+\gamma+\delta)} - Y \quad (30)$$

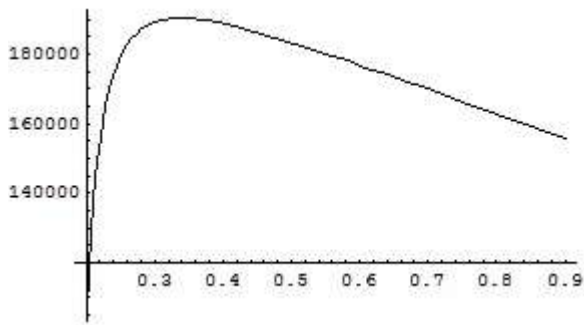
Now from equation (26), we will get the value of Q (t) and the value of p(t) is obtained from equation (3). The profit function  $\pi(\theta)$  can be obtained from equation (5). This profit function has maximum value if  $\frac{d\pi}{d\theta} = 0$  and  $\frac{d^2\pi}{d\theta^2} < 0$ .

#### 4. Numerical Examples

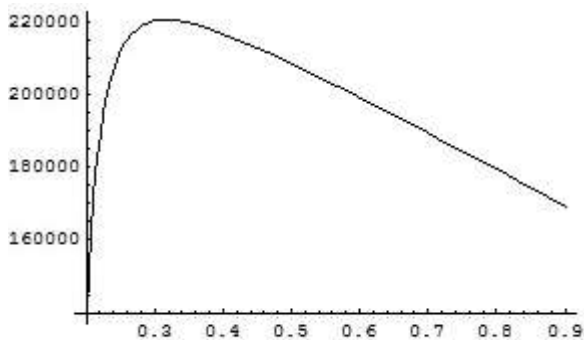
Example 1 (When demand rate is a quadratic function of time): Let the parameter values of some products are  $a = 200$ ,  $b = 10$ ,  $c = 5$ ,  $\alpha = \$0.01$ ,  $\gamma = 0.02$ ;  $L = 12$ ,  $r = \$0.16$ ;  $i = \$0.14$ ,  $A = \$200$ ,  $B = \$300$ ,  $C_r = \$25$ ;  $R = \$15$ ;  $W = \$80$ ;  $C_h = \$2.0$ ,  $\theta_{min} = 0.1$ ;  $\theta_{max} = 0.9$ ;  $k = 0.5$  in appropriate units. Then the optimal profit becomes  $\pi^* = \$214879$ . This maximum value is attained at  $\theta^* = 0.336714$ . The graph (Fig. 1, Annexure-1) shows that profit is concave function of  $\theta$ . So, the obtained solution is a global maximum solution.



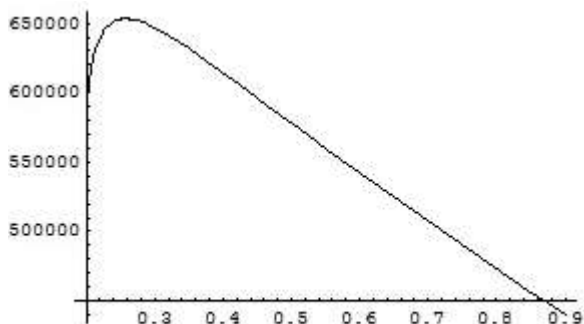
Example 2 (When demand rate is a linear function of time): Considering the values of all the cost and profit parameters as same as Example 1 but taking  $c=0$ , we get the maximum profit becomes  $\pi^* = \$190365$ . This maximum value is attained at  $\theta = 0.336714$ . The graph (Fig. 2, Annexure-2) shows that profit is concave function of  $\theta$ . So, the obtained solution is a global maximum solution.



Example 3 (When demand rate is exponential function of time): Let us consider the values of all cost and profit parameters same as Example 1 with exponential demand function  $f(t) = ae^{bt}$ , where  $a=100$  and  $b=0.25$ . Then, the maximum profit becomes  $= \pi * \$220450$ . This maximum value is attained at  $\theta=0.317$ . The graph (Fig. 3, Annexure-3) shows that profit is concave function of  $\theta$ . So, the obtained solution is a global maximum solution.



Example 4 (When demand rate is linear function of inventory): Let us consider the values of all cost and profit parameters same as Example 1 with linear demand function  $f(Q) = a + bQ$ , where  $a=200$  and  $b=5$ . Then, the maximum profit becomes  $= \pi * \$653183$ . This maximum value is attained at  $\theta=0.256909$ . The graph (Fig. 4, Annexure-4) shows that profit is concave function of  $\theta$ . So, the obtained solution is a global maximum solution.



## 5. Conclusion

While considering inventory models, two questions among the researchers have been growing interest,. One is deterioration and the other is the variation of demand function. When any firm or industry produces some products, they always try to manufacture all items of perfect quality. But instead of using all modern technology, sometimes industry or firm fails to produce all items of perfect quality. Sometimes defective



quality items are reworked at some cost to restore its quality to the original one. The rework cost can be minimized by reducing the product reliability parameter. But the reduction of product reliability parameter gives rise to the development cost of production. Higher development cost increases unit production cost. In the present model we have obtained the optimal value of the product reliability parameter. This ensures that the profit function has unique global maximum value at some optimal value of product reliability parameter.

This paper can be extended in many ways like considering time-varying deterioration on optimal policy or considering multi-item EOQ model.

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