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A Study of Weibull Distribution to Analyze the Wind Speed at Jogimatti in India

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Abstract: Wind is an available resource in nature that could be utilized by mechanically converting wind power into electricity using wind turbines. Wind energy is an indirect form of solar energy. Various probability distribution models were used for the statistical analysis of recorded wind speeds. This paper investigates the probability distributions of wind speed based on wind speed data recorded at JOGIMATTI station in INDIA. The Weibull distribution and Weibull-Weibull probability distribution function, Mixture Gamma and Weibull distribution, Mixture Normal and Weibull distribution and Maximum Endrophy distribution are adopted in this study to fit the wind speed data. It is found from the hypothesis test that Weibull distribution is more appropriate than the other distribution. This best fit probability distribution can be used to calculate the power density.A case study is given to discuss the probability analysis results.

Keywords: Wind speed , Moment method , Weibull Distribution , statistical analysis

1. Introduction

Towards the end of 20th and beginning of the 21st centuries, interest has risen in new and renewable energy sources especially wind energy for electricity generation. The scientists and researchers attempted to accelerate solutions for wind energy generation design parameters. Our life is directly related to energy and its consumption, and the issues of energy research are extremely important and highly sensitive.In a short time, wind energy is welcomed by society, industry and politics as a clean, practical, economical and environmentally friendly alternative. Wind energy has recently been applied in various industries, and it started to compete with other energy resources. Wind energy history, wind-power meteorology, the energy–climate relations, wind-turbine technology, wind economy, wind–hybrid applications and the current status of installed wind energy capacity all over the world reviewed critically with further enhancements and new research trend direction suggestions.

Wind energy can be utilized for a variety of functions ranging from windmills to pumping water and sailing boats.With increasing significance of environmental problems,clean energy generation becomes essential in every aspect of energy consumption. Wind energy is very clean but not persistent for long periods of time. In potential wind energy generation studies fossil fuels must be supplemented by wind energy. There are many scientific studies in wind energy domain, which have treated the problem with various approaches [1]–[3].General trends towards wind and other renewable energy resources increased after the energy crises of the 20th century [4].

Over 2000 year, water and windmills powered the world's first industries with new technology and materials.Modern wind turbines are used to generate the clean electricity needed for lighting, heating, refrigeration and other uses. Wind energy is a rather young industry, but one which already makes good economic sense. It is a proven success and its use is increasing and the downward trend in its costs is expected to continue. Most are operating in 'wind farms' as groups of wind turbines generating electricity on a significant scale. Single wind turbines are also being used for generating electricity, charging batteries, driving pumps and producing heat.

The physics behavior of wind shows great temporal and spatial variability. In meteorology, wind is air in motion, whose driving force is the uneven heating and cooling of the earth's surface. The horizontal movement of air parallel to the earth's surface is a measure of the wind in both direction and magnitude, which change most frequently. As a result, wind prediction is very difficult due to random change both in wind direction and speed. This changeability adds another measure importance to wind power. Wind power has unique characteristics for energy technology. The most significant impact on the environment is the visibility of wind turbines. Those who support the movement towards clean, sustainable energy production should take into consideration aesthetically pleasing symbols of a better future, especially when compared with the effects of acid rain, global climate change, radioactivity,land and water contamination in addition to other environmental problems associated with conventional energy sources.

As a meteorological variable, wind describes fuel of wind energy. In energy production, wind takes the same role as water, and wind variables should be analyzed. Windspeed deviation and changeability depend on time and area.This situation requires a new tendency for wind-speed modeling and search for the atmospheric boundary layer modeling as a special consideration in wind-power research. There are many papers concerning these subjects.

Wind turbines give great expectations for electricity generation possibilities. Convenient site selection for efficiency is important and necessary. Wind energy and speed change with time and are not continual at the same area during the whole year. Wind speed is a regionalized variable measured at a set of irregular sites.

Wind energy investigations mostly rely upon arithmetic average of the wind speed . However, many authors based the wind energy estimates on elaborated wind-speed statistics, including the standard deviation, skewness and kurtosis coefficient. Some researches advocate the use of two-parameter Weibull distribution in wind velocity applications . Their suggestions are taken as granted in many parts of the world for wind energy calculations.

2. Related Work

Generally, Weibull distribution function is used for representation of wind speed relative frequencies. Islam et al. [5] used two-parameter Weibull distribution function for wind speed forecasting and assessed wind energy potentiality at Kudat and Labuan, Malaysia. Celik [6] used Weibull-representative wind data instead of the measured data in time-series format for estimating the wind energy and had shown that estimated wind energy is highly accurately. Celik [7] made statistical analysis of wind data at southern region of Turkey and summarized that Weibull model was better than Rayleigh model in fitting the measured data distributions. Akdag et al. [8] discussed the suitability of two-parameter Weibull wind speed distribution and the two-component mixture Weibull distribution (WW-PDF) to estimate wind speed characteristics. Carta et al. [9] used WW-PDF. Because it is able to represent heterogeneous wind regimes in which there was evidence of bimodality or bitangentiality or, simply, unimodality.

Kiss and Imre [10] used Rayleigh, Weibull, and gamma distributions to model wind speeds both over land and sea. They found that generalized gamma distribution, which has independent shape parameters for both tails, provides an adequate and unified description almost everywhere. In recent past, mixture distributions were used to estimate wind energy potential that are quite accurate in describing wind speed characteristics. Jaramillo and Borja [11] used mixture Weibull distribution (WW) to model bimodal wind speed frequency distribution. Akpinar et al. [12] used mixture normal and Weibull distribution (NW), which is a mixture of truncated normal distribution, and conventional Weibull distribution to model wind speeds. Tian Pau [12] employed mixture gamma and Weibull distribution (GW) which is a combination of gamma and Weibull distributions . Also he employed Maximmum Endrophy distributions [14]for wind speed modeling.

3. Data Source

In this study mean wind speed datas for Jogimatti were collected from the meteorological society from 1989 to 2009 with mean average wind speed rating between 3.98 m/s to 8.33 m/s[Table I]. Our study includes the mean wind speed data for 12-months [Table II]observed at Jogimatti station in India. The wind speed measurement was by used of Cup-anemometer mounted at a height of 20m above the ground level. The data was then tabulated to daily and monthly mean wind speed frequency tables and analysed using MATLAB self developed programme and Microsoft Excel software.

Table1- Sample Wind Speed Data

Day	*v*	*Day*	*v*	*Day*	*v*	*Day*	*v*
1	13.0555	9	3.8888	17	7.5000	25	8.8888
2	8.0555	10	5.2777	18	8.0555	26	7.2222
3	6.6666	11	7.2222	19	5.8333	27	8.8888
4	10.2777	12	6.6666	20	3.8888	28	8.6611
5	7.2222	13	8.0555	21	5.0000	29	10.2777
6	6.6666	14	8.6111	22	3.6111	30	8.0555
7	8.0555	15	9.4444	23	8.8888	31	13.6666
8	5.0000	16	10.2777	24	4.4444		

Fig 1-Wind Gust Analysis For The Above Data

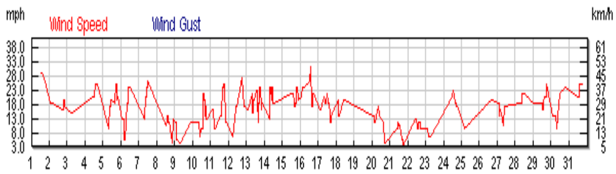

Table 2-Monthly Wind Speed Datas

Month	$\bar{v}$	*Month*	$\bar{v}$
Jan	6.0452	July	10.4671
Feb	5.7826	Aug	11.7204
Mar	6.6847	Sep	10.8460
April	7.5924	Oct	8.2645
May	7.1335	Nov	8.7144
June	8.3169	Dec	6.6211

4. Methods

In this paper Moment method is used to estimate the parameters of five selected distributions as Weibull, Weibull and Weibull pdf, Gamma and Weibull, Normal and Weibull and Maximum Endrophy Principle. The parameters were computed for each distribution and summarized in Table 3.

4.1 Weibull distribution (WD):

One of the most flexible distribution that can be used to represent various typical phenomena is Weibull. It is important to know that the no of hours per month or per year during which the given wind speeds occurred ie, the frequency distribution of the wind speeds. When the frequency percentage is plotted against the wind the frequency distribution emerges a curve. The top of this curve being the most frequent wind speed. This frequency distribution is used also to identify the most suitable site for the wind Turbine.

The Weibull distribution gives a good match with the experimental data. This is characterized by the shape parameter k and scale parameter c(m/s). The Weibull distribution is a continuous probability distribution. It is named after Waloddi Weibull who described it in detail in 1951 although it was first identified by Frchet in 1927 and first applied by Rosin and Rammler in 1933 to describe the size distribution of particles. The Weibull distribution is often used in the field of life data analysis due to its ability to fit the Gamma distribution and normal distribution and interpolate a range of shapes in between them.

The probability density function is given by

$$f(v) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} exp\left(-\left(\frac{v}{c}\right)^{k}\right) \quad (1)$$

And the cumulative distribution function is

$$F(v) = 1 - exp\left(-\left(\frac{v}{c}\right)^{k}\right) \quad (2)$$

Where v is the wind speed, k is the shape parameter and c is the scale parameter.

4.2 Weibull and Weibull PDF(WW):

In this study we considered a bimodal PDF to fit the measured data.WW-PDF also provides an analytical approaches to estimate the wind speed frequency distribution for JOGIMATTI . The WW-PDF is defined by

$$F_{ww}(v) = p[F_w(v)]_{left} + (1-p)[F_w(v)]_{right} \quad (3)$$

(or) in an explicit manner

$$F_{ww}(v) = p\int_0^{\infty} \frac{k_1}{c_1}\left(\frac{v}{c_1}\right)^{k_1-1} e^{\left(\frac{-v}{c_1}\right)^{k_1}} dv$$

$$+(1-p)\int_0^{\infty} \frac{k_2}{c_2}\left(\frac{v}{c_2}\right)^{k_2-1} e^{\left(\frac{-v}{c_2}\right)^{k_2}} dv = 1 \quad (4)$$

Where $F_{ww}(v)$ is the bimodal WW-PDF, v is the wind speed, c_1 and c_2 are the scale parameters, k_1 and k_2 are the shape parameters , p is the weight component of the Weibull distribution(0<p<1). T he weight component p can be obtained by using the following formulas

$$\bar{v} = p\bar{v}_1 + (1-p)\bar{v}_2 \quad (5)$$

and

$$\sigma^2 = p({\sigma_1}^2 - (p-1)(\bar{v}_1 - \bar{v}_2)^2) - (p-1){\sigma_2}^2 \quad (6)$$

where $\bar{v}$ is the average wind speed and σ is the standard deviation of the time series measured data , $\bar{v}_1$ and $\bar{v}_2$ are the average wind speed of the left and right Weibull distribution respectively. And ${\sigma_1}^2$ and ${\sigma_2}^2$ are the variance of the left and right Weibull distribution. The parameters c_1 , c_1, k_1 and k_2 can be obtained by solving the equations

$$\bar{v}_i = c_i\, \Gamma\left(1 + \frac{1}{k_i}\right)$$

and $${\sigma_i}^2 = {c_i}^2\left[\Gamma\left(1+\frac{2}{k_i}\right) - \Gamma^2\left(1+\frac{1}{k_i}\right)\right]$$

where i=1 for the left Weibull distribution and i=2 for the right Weibull distribution.

4.3 Mixture Gamma and Weibull Distribution(GW):

In Probability theory and statistics the Gamma distribution is a two parameter family of continuous probability distribution. It has a scale parameter λ and shape parameter α . If α is an integer then the distribution represents the sum of α independent exponentially distributed random variables each of which has mean λ . The Gamma distribution is frequently a probability model for waiting times; For instance in life testing the waiting time untill death is a random variable that is frequently modelled with a Gamma distribution.

The probability distribution function of Gamma distribution is expressed using the below function ,

$$g(v, \alpha, \beta) = \frac{v^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-v/\beta} \quad (7)$$

The cummulative Gamma distribution is given by

$$G(v, \alpha, \beta) = \int \frac{v^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-v/\beta}\, dv \quad (8)$$

The Generalized Gamma distribution presents a flexible family in the varieties of shapes and harard functions for modelling duration. It was introduced by Stacy[15]

The probability density function of the Mixture Gamma and Weibull distribution is given by

$$h(v; \alpha, \beta, k, c, w) = w\, g(v; \alpha, \beta) + (1-w)\, f(v; k, c) \quad (9)$$

And the cummulative distribution function is given by

$$H(v; \alpha, \beta, k, c, w) = w\, G(v; \alpha, \beta) + (1-w)\, F(v; k, c) \quad (10)$$

4.4 Mixture Normal and Weibull distribution(NW):

The Generalized Normal Distribution or Generalized Gaussian Distribution is either of paraametric continuous probability distributions on the real line. The Generalized Normal family includes the below well- known models as sub families Half normal distribution,Rayleigh distribution,Maxwell-Boltzmann distribution and Chi-Square distribution.

The probability density function of the Mixture distribution comprising of truncated normal and conventional weibull is written as

$$s(v; \mu, \sigma, k, c) = w\, q(v; \mu, \sigma) + (1-w) f(v; k, c) \quad (11)$$

Its cummulative distribution is given by

$$S(v; \mu, \sigma, k, c) = w\, Q(v; \mu, \sigma) + (1-w) F(v; k, c) \quad (12)$$

4.5 Maximum Entropy Principle Distribution (MEP):

For a given probability density function $f(x)$, its entropy is defined as :$H = -\int f(x) \ln f(x)\, dx$ (13)

Maximizing the entropy subject to some constraints enables one to find the most likely probability density function if the information available is limited. For the analysis of wind distribution, the classic solution of the maximum entropy problem can be written by:

$$f(v) = \exp\left[-\sum_{n=0}^{N} \lambda_n v^n\right] \quad (14)$$

Where λ_n are the Lagrange multipliers, which can be obtained by the standard Newton method; N is the number of moment constraints of a physical system, which generally lies between 3 and 6 in wind researches.

Table3: *The Distribution Parameters*

Distribution	*Parameters*		*Distribution*	*Parameters*	
Weibull	k	2.580110	NW	w	0.475591
	c	9.214211		μ	11.49039

			σ 2.472890 k 1.736901 c 5.701302
WW	w 0.454502 k_1 4.625902 c_1 12.132011 k_2 1.737302 c_2 5.157204	MEP	λ_0 5.100849 λ_1 -1.271330 λ_2 0.188583 λ_3 -0.011109 λ_4 0.000255
GW	w 0.439840 α 2.350691 β 1.960793 k 4.420010 c 11.950001		

5. Goodness Of Fit Test

A proper statistical analysis of wind data is a very important step in performing a wind resource assessment which provides a wind energy development initiative. Four tests were used to analyze the accuracy of the five distributions, namely Kolmogrov-smirnov Test, Root mean square value(RMSE) , Correlation Co-efficient (R^2) and Chi-quare (χ^2) tests. The Results of statistical tests are given in Table4.

5.1 Kolmogrov-smirnov Test(K-S test):

The K-S test (Chakravart, Laha and Roy 1967) is used to decide if a sample comes from a population with a specific distribution . K-s test is typically developed in the context of continuous distribution uncensored and ungrouped data. Now the K-S test statistically defined as

$$K - S = \max|c(v) - o(v)|$$

where c(v) and o(v) are the cummulative distribution functions for wind speed and observed wind speed data respectively. Lesser K-S value indicates better fit-ness of the PDE.

5.2 R^2 Test:

R^2 is a statistic that will give some information about the goodness of fit of a model which is statistically definrd as follows:

$$R^2 = \frac{\sum_{i=1}^{N}(y_i - z_i)^2 - \sum_{i=1}^{N}(y_i - x_i)^2}{\sum_{i=1}^{N}(y_i - z_i)^2}$$

Where N is the total number of intervals, y_i is the frequency of observation, x_i is the frequency of Weibull, z_i is the mean wind speed.

5.3 Chi-Square Error(χ^2 Error):

The Chi-square statistic is defined as

$$\chi^2 = \sum_{i=0}^{k}\frac{(O_i - E_i)^2}{E_i}$$

Where O_i is the observed frequency for bin i and E_i is the expected frequency for bin i, calculated by $E_i = F(x_2) - F(x_1)$ where F is the cumulative distribution function of the probability distribution being tested and x_1, x_2 are the limits for bin i.

5.4 Root Mean Square Value(RMSE):

The RMSE is calculated using the following relation

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}(y_i - x_i)^2\right]^{\frac{1}{2}}$$

Where N is the total number of intervals, y_i is the frequency of observation, x_i is the frequency of Weibull.

Table 4: Statistical Analysis

Distributions	*K-S error*	R^2	χ^2	*RMSE*
Weibull	0.0086	0.9999	0.0000	0.0018
WW	0.0112	0.9997	0.0010	0.0120
GW	0.0233	0.9997	0.0221	0.0021
NW	0.0114	0.9986	0.0001	0.0024
MEP	0.0107	0.9994	0.0172	0.0048

6. Result And Discussion

Weibull p.d.f perform better than the other distributions (Table5) .The statistical parameters for fitness evaluation of PDFs currently analyzed are presented inTable 4. It clearly shows that , considering K-S error, χ^2 and RMSE , weibull has the smallest error. If R^2 is considered Weibull has a value close to 1, which again conforms the Weibull p.d.f is suitable distribution for JOGIMATTI station.Wind frequency Graph resembles familiar bell-shaped curve; hence, Weibull PDF fits the observed distribution well [Fig 2].

Table 5 Result

Distribution	***Priority order***
Weibull	1
WW	3
GW	5
NW	4
MEP	2

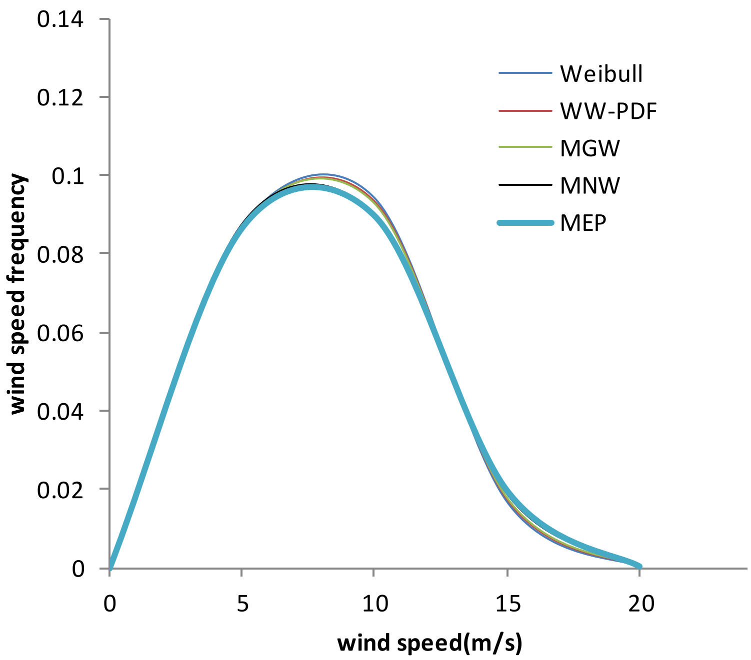

Fig-2

Conclusion

The wind energy resource potentials of JOGIMATTI station has been analysed statistically using the above discussed methods. Based on the analysis it is concuded that Weibull distribution fits to calculate the wind speed compare with other distributions.

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